Nonlinear Soil-Structure Interaction with Base Uplift *)

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SUMMARY

Buildings of large mass that have to be designed for severe earthquake excitation (e.g. buildings used in nuclear power plants) are often subjected to very large overturning moments. As a consequence of this load condition the base slab may partially uplift from the ground, and the building has to be designed for this situation.

For structures with approximately rigid foundations, soil-structure interaction can be described by a simple spring-dashpot system. The characteristics of these springs and dashpots are a function of the contact area between the foundation and the ground, and hence are nonlinear when uplift of the building occurs. Modal analysis is used, and the nonlinearities are accounted for by a modification of the load vector. For localized nonlinearities this method results in a simple and very efficient analysis. The coupling between vertical and rocking motion induced by the nonlinearity of the problem is considered.

The stress distribution under the base mat is needed to determine uplift. A linear stress distribution and the static stress distribution under a rigid strip on an elastic half-space are adopted in this investigation.

Results are given for a rectangular switch gear building of a nuclear power plant. Frequency independent dashpots were used for the geometrical damping of the soil. For a linear analysis the results agreed well with a complex response analysis in the frequency domain, in which the stiffness and damping were frequency dependent. When the nonlinearity of the problem is considered, the analysis yields slightly reduced results for the peak structural response (base moment, base shear and uplift). The peak structural response is sensitive to the amount of damping used. The stress distribution under the base mat has little effect on base moment and base shear, but a strong effect on uplift. Coupling between vertical and rocking motion becomes important for severe uplift conditions. Simultaneous vertical excitation has only a small effect on uplift.

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INTRODUCTION

A structure may show partial uplift of the ground when subjected to very strong earthquake shaking. The separation of the base mat from the soil constitutes a geometric nonlinearity which will affect the amplitude and the frequency content of the structural response. Several investigators have looked at the uplift problem. Wolf /5/ considered a circular reactor building. In his elaborate analysis the total foundation area was subdivided into elements and, assuming an elastic half-space, a full stiffness matrix was set up which related the contact forces and the displacements of the elements. The element forces were monitored, and tensile forces were not allowed. This approach is quite involved and often too time consuming because in most practical applications variations of earthquake motion, soil behaviour and structural properties have to be considered. In addition, the actual contact stress distribution under the base mat is extremely difficult to determine. It depends not only on the gross structural properties, the soil stiffness, and the magnitude of the ground motion, but also on the frequency of the motion, the flexibility of the base mat, nonlinear soil behaviour, pore water pressures in the soil (if applicable), and other conditions. Therefore, assumptions on the stress distribution have been used to simplify the problem, and discrete nonlinear springs and dashpots were determined to simulate the interaction of a rigid base mat and the soil /2, 3, 5, 6/.

In the present paper, the effects of damping, of simultaneous horizontal and vertical excitation, and of the assumed stress distribution in the contact area are investigated for a stiff nuclear power plant structure, which is relatively long compared to its width. The nonlinearity is restricted to the soil-structure interface. Therefore, the equations of motion can be efficiently solved using the normal modes of a linear system and adding correction forces to the external loads in order to account for the nonlinearity of the soil springs and dashpots.

2. Nonlinear interaction model

2.1 Uplift criterion and contact area.

Uplift occurs - by definition in this study - when and where the normal compressive stress under the base mat drops to zero. Tensile stresses are not permitted between the base mat and the soil. For a given base moment M and a vertical force V, the contact length $2\cdot\tilde{b}$ can be obtained simply from equilibrium conditions if a stress distribution under the base is assumed. The assumption of a linear distribution yields the uplift condition $|M|>V\cdot b/3$, and the contact length $2\cdot\tilde{b} = 3\cdot(b-|M|/V)$.

A more realistic assumption on the stress distribution under a rigid rectangular foundation is the static stress distribution obtained for a smooth rigid strip on a semi-infinite elastic half-space /1/:

$$\sigma_z = \frac{p}{\pi\cdot\tilde{b}} \cdot \frac{1}{\sqrt{1-(x/\tilde{b})^2}} \left(1 + 2 \cdot \frac{x}{\tilde{b}^2} \cdot \tilde{e} \right) \qquad (1)$$

in which p = vertical line load, $\tilde{b}$ = half width of the contact area, $\tilde{e}$ = eccentricity of the line load with respect to the center of the contact area. When this stress distribution is assumed uplift will occur for $|M|>V\cdot b/2$. Towards the edge $(x\to\tilde{b})$ the stress according to eq.1 approaches infinity. In reality, however, the stress value at the edge will be limited because the soil will yield locally. Analytically, this can be described by a yield stress factor $\tilde{\alpha}$

and a yield coordinate x_y (see Fig. 1):

$$\tilde{\alpha} = \frac{2\cdot\sigma y\cdot\tilde{b}}{p} ; \quad \frac{x_y}{\tilde{b}} = \zeta = \frac{\pi^2\cdot\tilde{\alpha}^2 - 4}{\pi^2\cdot\tilde{\alpha}^2 + 4} \quad (2)$$

Integration of the stress distribution shown in Fig. 1 yields the eccentricity $\tilde{e}$ of the line load:

$$\frac{\tilde{e}}{2\cdot\tilde{b}} = \frac{\arcsin\zeta + \pi/2 - (2+\zeta)\cdot\sqrt{1-\zeta^2} + \tilde{\alpha}(1-\zeta^2)\cdot\pi/2}{4\cdot\arcsin\zeta + 2\pi - 4\cdot\sqrt{1-\zeta^2} + 2\tilde{\alpha}\,(1-\zeta)\cdot\pi} \quad (3)$$

The uplift condition is then given by $|M| > V\cdot\tilde{e}$, and one-half the contact length is $\tilde{b} = b + \tilde{e} - |M|/V$. It has to be noted that $\tilde{e}$, M and V depend on $\tilde{b}$ and have to be determined iteratively.

2.2 Soil-structure interaction forces

The soil is assumed to behave as a linear viscoelastic half-space. The soil springs and dampers for the rectangular foundation 2a x 2b are approximated by those of an equivalent circular foundation /4/:

$$k_z = \frac{4\cdot G\cdot r_z}{1-\nu} \qquad c_z = 0.85 \cdot \frac{k_z\cdot r_z}{V_s}$$

$$k_\phi = \frac{8\cdot G\cdot (r_\phi)^3}{3(1-\nu)} \qquad c_\phi = \frac{0.3\cdot a_o^2}{1 + a_o^2} \cdot \frac{k_\phi\cdot r_\phi}{V_s}$$

$$k_x = \frac{8\cdot G\cdot r_x}{2-\nu} \qquad c_x = 0.576 \cdot \frac{k_x\cdot r_x}{V_s} \quad (4)$$

with G as shear modulus, ν as Poisson's and V_s as shear wave velocity of the soil, and the equivalent radii $r_z = r_x = \sqrt{4\cdot a\cdot b/\pi}$; $r_\phi = \sqrt[4]{16a\cdot b^3/(3\pi)}$

The damping coefficient c_ϕ depends on the frequency ratio $a_o = 2\pi f\cdot r_\phi/V_s$. To obtain a frequency independent damper, the frequency f is taken to be approximately the fundamental rocking frequency of the system.

In case of uplift the contact area is reduced, and thus the soil stiffness and damping decreases. The reduced spring and damper constants are obtained from eqns. (4) if the foundation width 2b is substituted by the contact length $2\tilde{b}$ according to section 2.1. They act in the center of the contact area and must be transformed to the center of the base slab, which results in a coupling of vertical and rocking motion (Fig. 2). The soil structure interaction forces refering to the center of the base slab are obtained with the reduced stiffness and damping constants, denoted by ~, as:

$$V = \tilde{k}_z \cdot (w + |\phi|\cdot(b-\tilde{b})\ ) + \tilde{c}_z \cdot (\dot{w} + \dot{\phi} \cdot (b-\tilde{b}) \cdot \mathrm{sign}(\phi)) \quad (5a)$$

$$M = \tilde{k}_\phi \cdot \phi + \tilde{c}_\phi \cdot \dot{\phi} + V \cdot (b-\tilde{b}) \cdot \mathrm{sign}(\phi) \quad (5b)$$

$$H = \tilde{k}_x \cdot u + \tilde{c}_x \cdot \dot{u} \quad (5c)$$

In case of full contact the notation ~ is omitted. In this case $(b-\tilde{b}) = 0$ and the rocking and vertical motions are uncoupled.

The stiffness and damping coefficients in eqns. 5 are based on the assumption of a rigid plate on a linear elastic half-space (eqns.4). This assumption implies a stress distribution under the base mat which is not quite consistent with those used in the determination of the contact ratio. However, the stress distribution under the rigid plate is bounded by those which are actually used, i.e., by the linear and the rigid strip distribution.

3. Method of analysis

A beam model is analyzed in which nonlinear soil-structure interaction forces as described in section 2 are considered. For this system the equations of motion, including the degrees of freedom at the foundation, can be written as:

$$\underline{M}_s \cdot \ddot{\underline{u}} + (\underline{K}_s + \underline{K}_f) \cdot \underline{u} = -\underline{M}_s \cdot (\underline{I}_x \cdot \ddot{u}_{gx} + \underline{I}_z \cdot \ddot{u}_{gz}) - \tilde{\underline{P}} + \underline{K}_f \cdot \tilde{\underline{u}}_f \qquad (6)$$

$$\underline{u}_f = \begin{Bmatrix} w \\ \phi \\ u \end{Bmatrix} \quad \underline{u} = \begin{Bmatrix} \underline{u}_s \\ \hline \underline{u}_f \end{Bmatrix} \quad \tilde{\underline{P}} = \begin{Bmatrix} \underline{o} \\ \hline V-W \\ M \\ H \end{Bmatrix} \quad \underline{K}_f = \left[\begin{array}{c|ccc} \underline{o} & & \underline{o} & \\ \hline \underline{o} & k_z & & \\ & & k_\phi & \\ & & & k_x \end{array}\right] \quad \tilde{\underline{u}}_f = \begin{Bmatrix} \underline{o} \\ \hline \underline{u}_f \end{Bmatrix}$$

with

$\underline{M}_s$, $\underline{K}_s$ = Mass and stiffness matrix of structure, respectively.

$\underline{u}_s$ = Vector of relative structural displacements excluding foundation DOFs.

$\underline{I}_x$, $\underline{I}_z$, = Influence coefficient vectors; all elements corresponding to degrees of freedom in x (I_x) and the z (I_z) directions are 1, all other elements are 0.

$\ddot{u}_{gx}$, $\ddot{u}_{gz}$ = Ground acceleration time histories in the x and z directions, resp.

W = Weight of structure

The nonlinear uplift effect is taken into account by correction forces on the right side of the equations.

The solution of the homogeneous equations are the eigenvectors and eigenvalues of the linear system without uplift. By use of this eigensolution, the inhomogeneous equations are transformed to normal coordinates. The time dependent normal-coordinate-equations must be integrated simultaneously since the correction forces depend on the time history of the displacements w, ϕ, and u. An explicit integration method (central difference) is used. The restriction on the size of the time step Δt to assure the stability of the method ($\Delta t < T_n/\pi$, T_n = period of n-th mode) is easy to satisfy because only a few modes are necessary to describe sufficiently accurate the structural response. Modal damping is included in the analysis to take into account the material damping of structure and soil.

4. Numerical Results

As a case study a switch gear building of a nuclear power plant is considered. The building has a rectangular base mat (28.9m by 76.8m), a structural weight of 533 MN, and the center of gravity is 14.15m above the base. The subgrade consists of gravel and sand with an average shear modulus G= 140 MN/m² and Poisson's ratio ν = 0.47. The model used in the analysis is shown in Fig. 3. The natural frequencies of the 5 lowest modes for the linear case without uplift are 2.1, 2.9 (vertical), 5.6, 19.5 and 21.1 Hz.

Only ground motion in the plane normal to the long axis of the building is considered. The vertical and horizontal earthquake motions are assumed to be uncorrelated. Three different time histories of ground acceleration with similar response spectra are used. One of the time histories and its response spectrum are shown in Fig. 4.

Three different methods of analysis are applied:

A: Linear complex response analysis in the frequency domain, in which the frequency dependence of the soil springs and dashpots is included, and Fast Fourier Transformations are used to compute the response in the time domain.

B: Modal time history analysis with soil springs and dashpots which are independent of frequency but depend on the contact area, as described in section 2.2.

C: The same as method B but with modal damping independent of the contact area as the only form of energy dissipation. Modal damping values are strain energy weighted averages of the damping associated with the structure and the soil. The modal damping calculated has been limited to a maximum of 15% for horizontal and 30% for vertical motion in accordance with /7/.

It has been found for the time integration involved in methods B and C that sufficiently accurate results are obtained with the inclusion of 5 modes. A time step of 0.001 s with the central difference scheme resulted in an accurate solution of the modal equations. The cost associated with such a small time step is negligible for the example used.

Table I shows, for one combination of horizontal and vertical ground motion, results obtained by the three different methods. The yield stress according to section 2.1 is given as a normalized parameter $\alpha = \sigma_y \cdot 4ab/W$. In the linear case (no uplift), the results of method B agree well with those of method A. However, the results of method C, in which the damping is limited, overestimates both the base shear and, in particular, the base moment. As can be seen, the response (base shear and moment) is reduced in the nonlinear case by the uplift. The assumption of a particular stress distribution has a considerable effect on the contact ratio, but has little effect on the structural response.

Fig. 5 shows response time histories for a strong excitation. Fig. 6 shows three time histories of the contact ratio for different assumptions of the stress distribution and levels of excitation. It illustrates that not only the amount of uplift but also the frequency of occurrence and duration of uplift are influenced considerably by the stress distribution and by the maximum ground acceleration.

Fig. 7 gives the maximum base moment and contact ratio as functions of the maximum ground acceleration for different stress distributions. The assumed stress distribution has a strong influence on the excitation level at which uplift is predicted to start. Only a moderate reduction of the maximum base moment is caused by uplift. However, the moment at the center of the contact area is reduced considerably because of the significant reduction in the rocking stiffness. The contribution of the eccentric vertical force to the moment at the center of the base mat - see Fig. 2 - becomes increasingly important as the contact ratio decreases. This coupling effect between vertical and rocking motion in a nonlinear analysis produces another interesting result. Even with no vertical ground excitation, the structure is excited vertically. At the instant of maximum uplift the static vertical force is reduced by about 14% of the maximum horizontal ground acceleration, which is equivalent to .07g for $a_x = 0.5g$. However, in most practical applications the contribution of this coupling effect to the vertical force can be neglected because it is small in comparison to the vertical force which results from the vertical excitation.

The influence on the structural response of three different time histories for the horizontal ground acceleration was investigated. The influence was also investigated for 18 different combinations of the horizontal and vertical ground acceleration. The results are given in Table II.

Additional Considerations and Conclusions

Sliding of the structure has not been considered in this study. It was verified that for the soil conditions and maximum levels of excitation investigated sliding would not occur if the effective vertical force is not reduced considerably by the buoyancy. The influence of the buoyancy on uplift is an important aspect for many practical applications. Some investigators (e.g. /2/) have subtracted the buoyancy from the weight to determine the effective vertical force. This is a conservative assumption because it means that a complete pore water redistribution has to take place during the very short duration of uplift. It appears that for many soil conditions this assumption is too conservative. The presented analyses have been carried out using total stresses. It should be pointed out that not all effects present in the physical system are included in this model. In particular, the frequency dependency of the soil parameters is neglected in the nonlinear analysis.

Finally, the following conclusions can be drawn:

a) Nonlinear earthquake response analyses with consideration of partial base mat uplift yield smaller values of maximum base shear and overturning moment than linear analyses without consideration of uplift. In the investigated case the maximum reduction of the overturning moment is approximately 15%.

b) Damping has a strong effect on the computed structural response.

c) The assumed stress distribution under the base mat has little effect on the peak structural response (acceleration, base shear, overturning moment). Yet, it does affect the computed contact area, and consequently the soil springs and dashpots.

d) Simultaneous vertical excitation has only a small effect on the contact ratio. The peak overturning moment may be combined with the static vertical force reduced by 1/3 of the peak dynamic vertical force, which is caused by the vertical earthquake excitation, to compute the maximum eccentricity.

e) Floor response spectra obtained by a linear and nonlinear analysis have been compared. For the example considered here the spectra for the two cases were found to be virtually identical.

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TYPE OF ANALYSIS	linear neglecting uplift		nonlinear including uplift			
	$\frac{H}{W}$	$\frac{M}{W \cdot c}$	stress [2] distr.	$\frac{H}{W}$	$\frac{M}{W \cdot c}$	min $\frac{\tilde{b}}{b}$
A) Frequency domain	.522	.768	-	-	-	-
B) Frequency independent soil springs and dashpots	.525	.781	T	.478	.652	.51
	-	-	R, α= 8	.477	.670	.65
	-	-	R, α=16	.482	.674	.67
C) Modal damping 1)	.595	.923	T	.480	.699	.44

1) Damping ratios of horizontal modes: .15; .15; .05; .05; / of vertical mode: .30
2) T = triangular distribution; R = rigid strip distribution
$\alpha = \sigma_y \cdot 4ab/W$ (yield stress / average normal stress under base mat)

Table I: Maximum base shear H and base moment M for ground motion with a_x=0.5g and a_z=0.25g

Several time histories with spectra similar to that in Fig. 4	a_z = 0.			a_z = .25g		
	$\frac{H}{W}$	$\frac{M}{W \cdot c}$	min $\frac{\tilde{b}}{b}$	$\frac{H}{W}$	$\frac{M}{W \cdot c}$	min $\frac{\tilde{b}}{b}$
mean value	.493	.653	.61	.495	.663	.59
standard deviation	.036	.009	.03	.032	.012	.03

Table II: Maximum base shear H, maximum base moment M, and minimum contact ratio for several time histories, a_x = 0.5g, α = 8

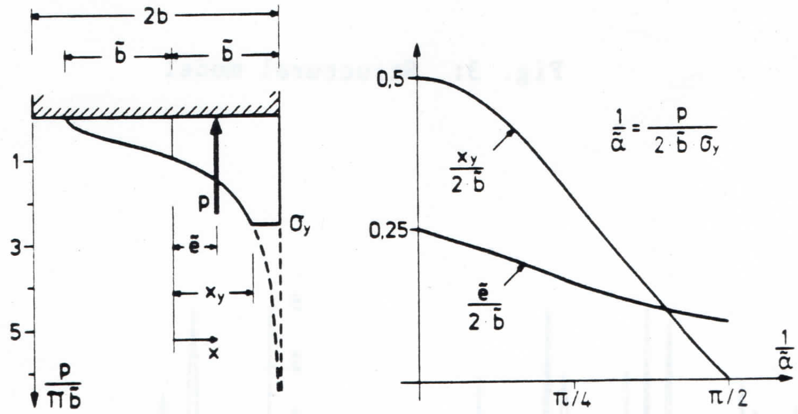

Fig. 1: Stress distribution σ_z, eccentricity $\tilde{e}$ of stress resultant and yield coordinate x_y

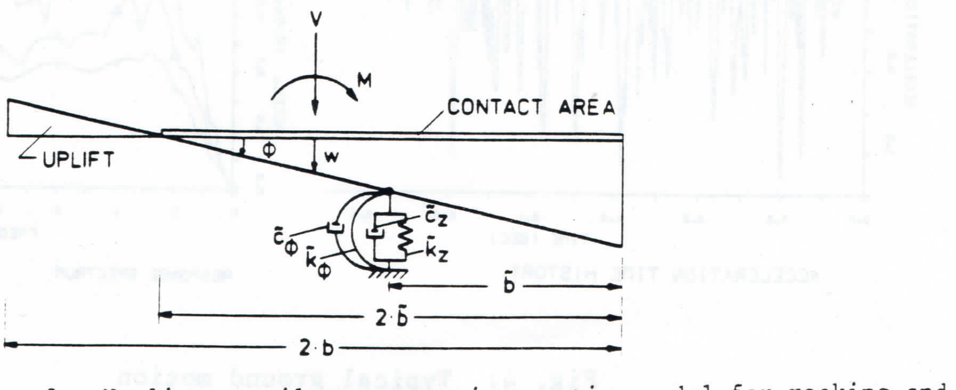

Fig. 2: Nonlinear soil-structure interaction model for rocking and vertical motion (horizontal springs and dashpots not shown)

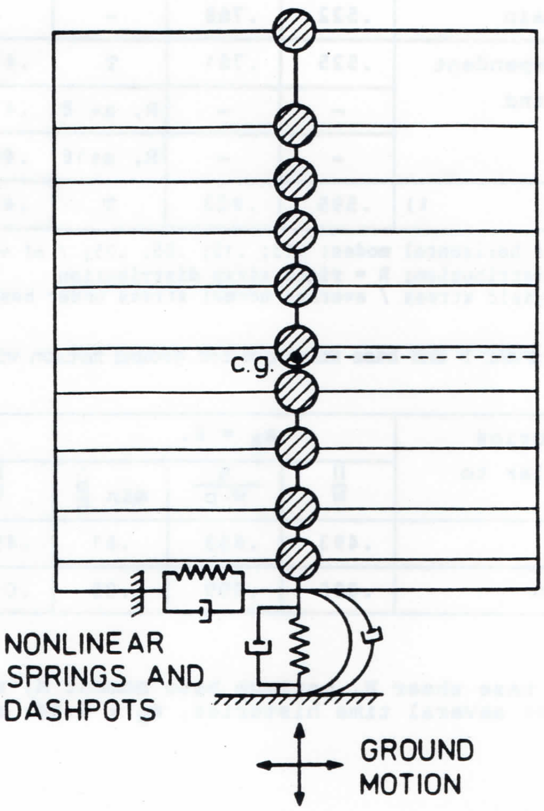

Fig. 3: Structural model

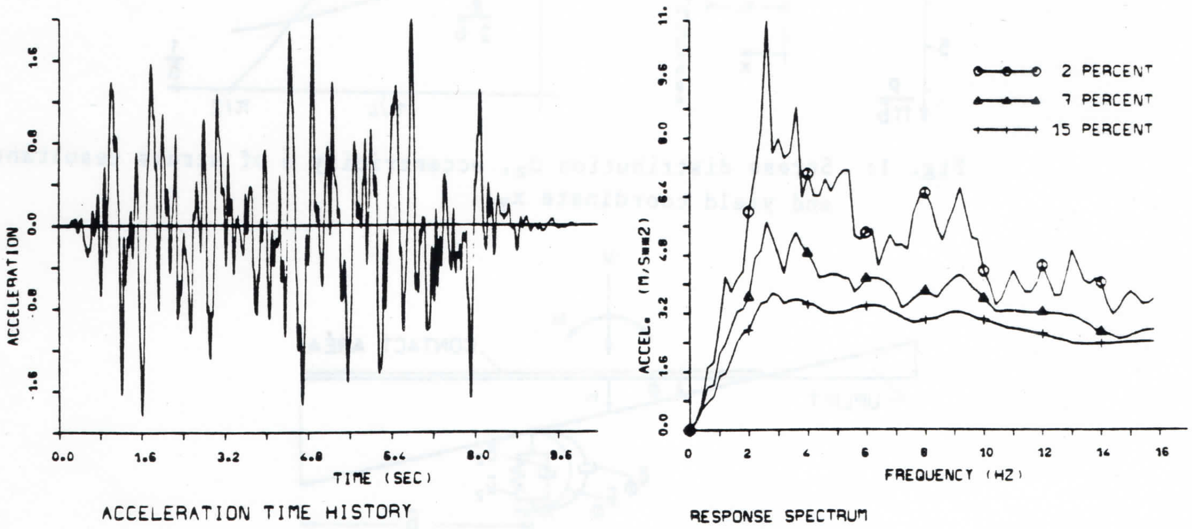

Fig. 4: Typical ground motion

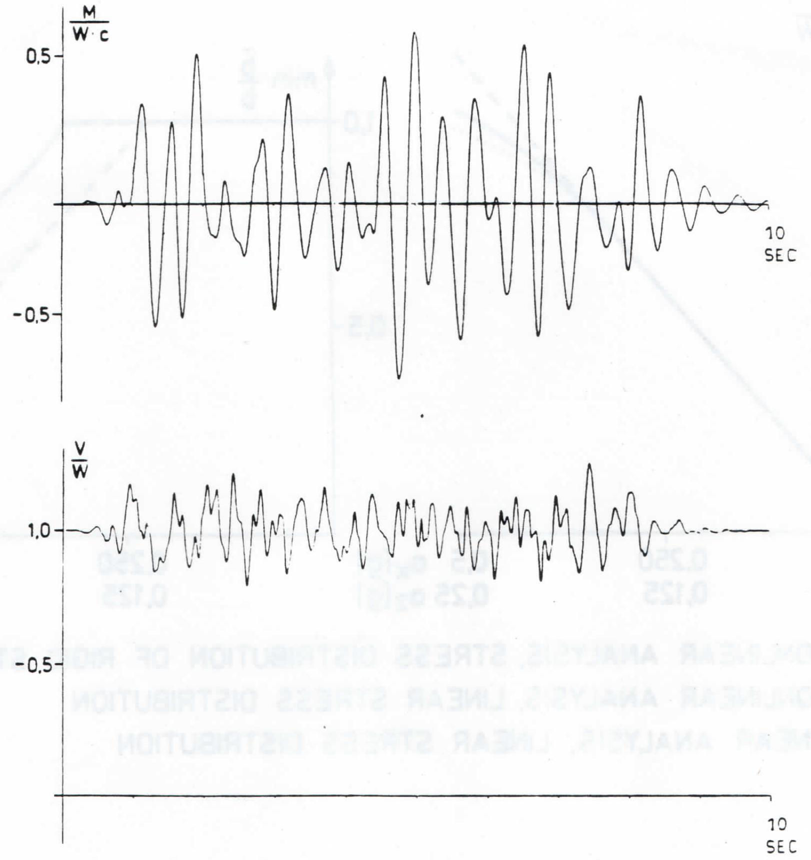

Fig. 5: Base moment and vertical force, $a_x = 0.5g$ $a_z = 0.25g$, $\alpha = 8$

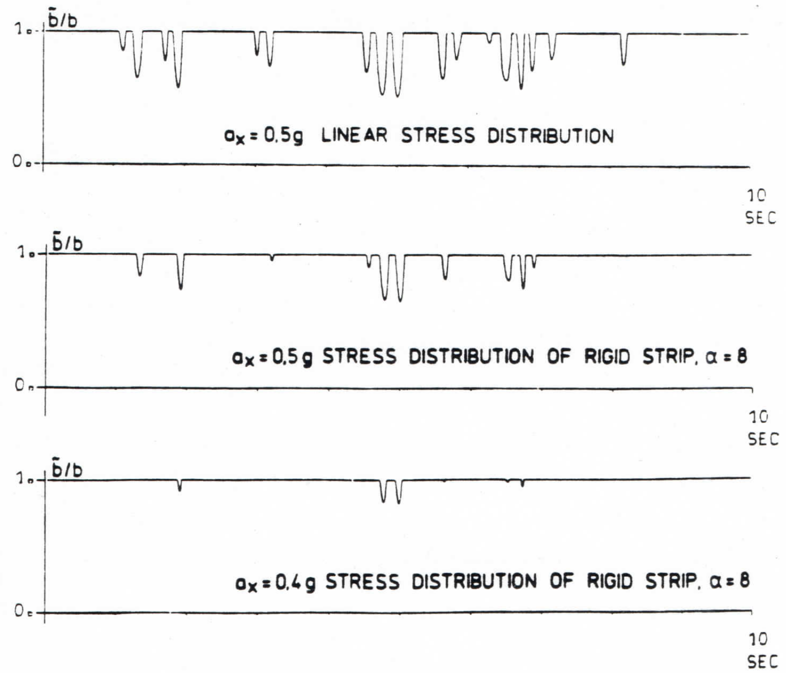

Fig. 6: Time histories of contact ratio, $a_z = a_x/2$

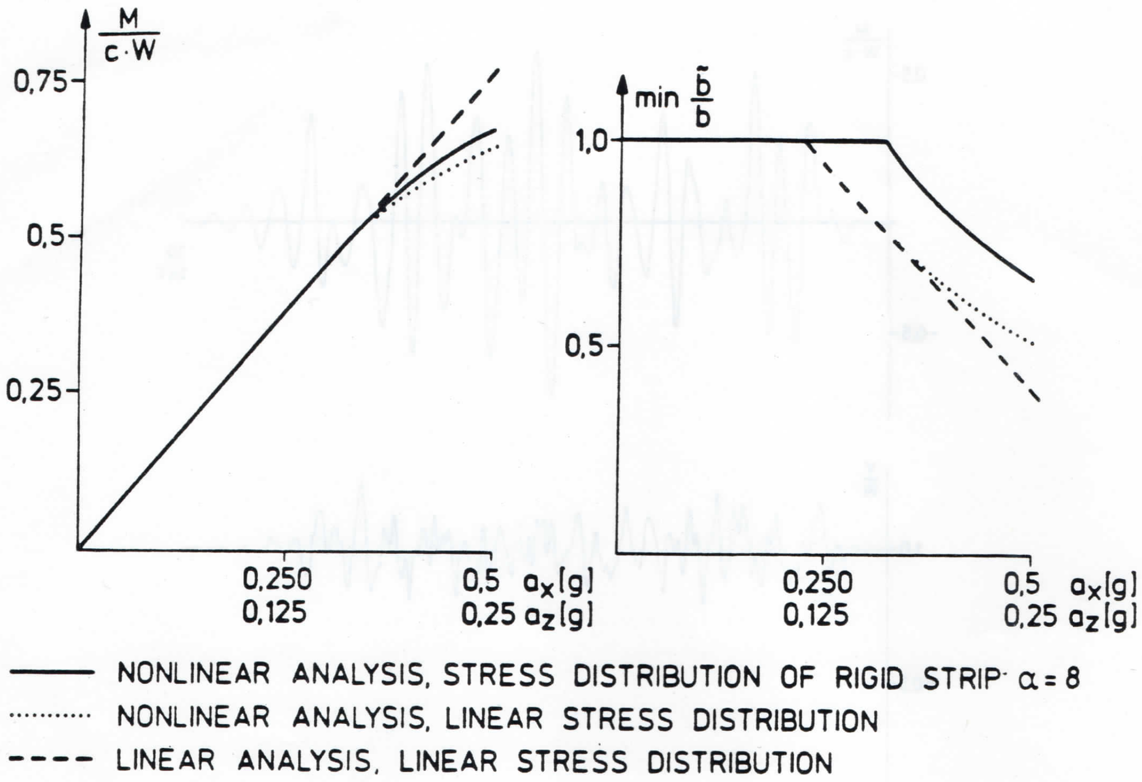

Fig. 7: Maximum base moment M and minimum contact ratio $\tilde{b}/b$