DYNAMIC STIFFNESS OF FOUNDATIONS ON INHOMOGENEOUS SOILS *

H. Werkle I)
G. Waas

SUMMARY

The dynamic stiffness of a rigid circular foundation on a viscoelastic soil layer with linearly increasing shear modulus is investigated. The equations of motion of the soil are solved in the frequency domain by a semi-analytical method. The increase rate of the shear modulus as well as the Poisson ratio of the soil are varied. A reduction in damping is found to result from inhomogeneity. It is shown that the static stiffness as well as the frequency-dependent stiffness and damping coefficients of a soil with linearly increasing shear modulus can be well approximated by those of an elastic halfspace using a different representative shear modulus and transformation of the frequency variable for each vibration mode of the foundation.

INTRODUCTION

In earthquake analysis of structures the foundation stiffness is generally represented by springs and dampers. It is often computed using the simplifying assumption of a rigid circular base on a homogeneous elastic halfspace. The dynamic stiffness for this very basic case is well known /1, 2, 3/. In reality, however, soils are rarely homogeneous. In many cases the stiffness of cohesive and cohesionless soils increases with overburden and thus with depth. To cope with this situation one tries to find an equivalent homogeneous halfspace using a "representative" shear modulus for the soil.

A simple method for selecting its value is to take the modulus of the soil at a "representative" depth Z_r. This depth is often picked more or less by intuition (e.g. at a depth equal to half a foundation radius) without concern for differences between the various vibration modes, i.e. vertical, horizontal, rocking, torsional motion. The application of this approximation is questionable.

A more appropriate method has been suggested by Holzlöhner /4/. He recommends an averaging procedure for the determination of a representative shear modulus using as weighting factor the relative contribution of each depth interval to the total displacement of the foundation. His results indicate that different values of the shear modulus should be used for different vibration modes and that radiation damping is overestimated if the modulus appropriate for the static stiffness is also used to evaluate the damping ratio.

In the following, numerical results are presented for a rigid circular foundation on a soil layer with shear modulus increasing linearly with depth. The layer thickness is chosen very large so that its bottom boundary has virtually no effect on the foundation response. Based on the parameter study, a procedure is suggested for selecting the representative moduli which can be used with the stiffness and damping formula for a homogeneous halfspace.

I) Hochtief AG, Abt. KTI, 6000 Frankfurt am Main, W.-Germany

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METHOD OF ANALYSIS

The soil is assumed to be a viscoelastic, horizontally layered continuum. It extends laterally to infinity and is bounded in the vertical direction by a rigid base at a finite depth. The determination of the soil stiffness is based on a semi-analytic solution for axisymmetric and non-axisymmetric ring loads. A detailed derivation is presented in Ref. 5. A Fourier expansion is used for the variation in the tangential direction. To describe the stiffness of a rigid foundation, only the terms n = o (vertical and torsion) and n = 1 (horizontal and rocking) are needed. The displacement field is approximated by shape functions with a piecewise linear variation in vertical direction and by analytic solutions of the equations of motion to describe the variation in the horizontal direction. Observing homogeneous boundary conditions (zero stress at the surface and zero displacements at the base), yields an algebraic eigenvalue problem. The eigenmodes are generalized Rayleigh and Love waves, and the eigenvalues are the corresponding wave numbers. The displacements, strains, and stresses of the axisymmetric system are expanded in terms of the eigensolutions. Application to the case of a ring load acting within or on the surface of the layered soil yields an explicit solution for the displacement field.

The dynamic flexibility matrix of the nodal rings, at which the soil is to be connected to the foundation, contains the nodal displacements that are produced by unit loads applied at each of the nodal rings. Inversion of the flexibility matrix yields the complex dynamic stiffness matrix. It is transformed to the vibration modes of a rigid circular foundation. In the following only the diagonal elements of the resulting stiffness matrix - denoted by "vertical", "horizontal", "rocking" and "torsion" - will be considered.

SOIL MODEL

The shear modulus G increases linearly with depth Z as

$$G(Z) = G_o \left(1 + \alpha \frac{Z}{R}\right) \tag{1}$$

where G_o is the modulus at the soil surface, R the radius of the foundation and α a coefficient for the increase rate. The soil model has a thickness of $H = 10 \cdot R$, which is sufficient for the simulation of a halfspace /6/. Its discretization is shown in fig. 1.

The static spring constants computed for a foundation on a homogeneous layer ($\alpha = o$) compares well with those for a homogeneous halfspace /1, 2, 3/. Differences are less than 5% in the rocking, horizontal and torsional modes, and 12% in the vertical mode.

The cases $\alpha = o; 1; 2$ with Poisson ratio $\nu = 0.2; 1/3; 0.45$ were investigated. The material damping ratio was set to $D = 0.05$.

NUMERICAL RESULTS

Notation for stiffness and damping

The stiffness of the foundation in any mode may be expressed in the frequency domain by

$$\tilde{K} = K \left(k \cdot (1 + 2iD) + i \, \tilde{a} \, c \right) \tag{2}$$

in which K is the static stiffness constant, and k and c are frequency dependent coefficients for stiffness and radiation damping, respectively. The variable

$$\tilde{a} = \frac{\omega . R}{\sqrt{\tilde{G}/\rho}} \tag{3}$$

represents the related, dimensionless frequency. ω is the circular frequency of the vibration and ρ the mass density of the soil. $\tilde{G}$ denotes an equivalent shear modulus discussed later.

Material damping of the soil is simulated by a complex modulus $G\cdot(1+i2D)$ in which D is the hysteretic damping ratio. For a single degree of freedom system in resonance one may combine radiation and hysteretic damping to an equivalent modal damping by

$$\xi = \frac{\tilde{a}}{2}\frac{c}{k} + D \tag{4}$$

Static stiffness

The static spring constants K can be expressed by the formula's of an homogeneous halfspace (α = o):

$$\frac{4\ \bar{G}\ R}{1-\nu} \qquad \text{vertical} \tag{5a}$$

$$\frac{8\ \bar{G}\ R}{2-\nu} \qquad \text{horizontal} \tag{5b}$$

$$\frac{8\ \bar{G}\ R^3}{3(1-\nu)} \qquad \text{rocking} \tag{5c}$$

$$\frac{16\ \bar{G}\ R^3}{3} \qquad \text{torsion} \tag{5c}$$

For each mode a different equivalent shear modulus $\bar{G} = G_o\cdot(1+\alpha\zeta)$ is chosen, which corresponds to a representative depth $\bar{Z}$ according to eq. (1). For the cases analysed, the dimensionless depths

$$\zeta = \bar{Z}/R \tag{6}$$

are given in table I. The value of ζ is approximately invariant with respect to α, ν, but different for the various modes:

$$\zeta \approx \begin{cases} 1 & \text{vertical} \\ 0.5 & \text{horizontal} \\ 0.4 & \text{rocking} \\ 0.2 & \text{torsion} \end{cases}$$

Dynamic coefficients

The frequency dependent coefficients c and k for α = 0, 1, 2 and ν = 1/3 are shown in fig. 2. They are plotted over the dimensionless frequency $\tilde{a}$. This frequency is computed by eq. 3 using the shear modulus $\tilde{G} = G_o\ (1 + \alpha\tilde{\zeta})$ at the depth $\tilde{Z} = \tilde{\zeta}\cdot R$. The value $\tilde{\zeta}$ depends on frequency and can be set to, fig. 3:

$$\tilde{\zeta} = \delta \cdot \lambda_o/R < 10\ \delta \tag{7}$$

with

$$\lambda_o = 2\pi\cdot\sqrt{G_o/\rho}\ /\ \omega$$

and

$$\delta = \begin{cases} 1.5 & \text{vertical} \\ 0.75 & \text{horizontal} \\ 0.75 & \text{rocking} \\ 0.25 & \text{torsion.} \end{cases}$$

The value λ_o may be understood as the shear wave length in a homogeneous soil with the shear modulus G_o. The use of the dimensionless frequency $\tilde{a}$ instead of $\overline{a} = \omega \cdot R/\sqrt{\overline{G}/\rho}$ (related to the statically equivalent halfspace) corresponds to a frequency dependent modification of the scale of the frequency axis.

With this procedure, the damping coefficients c for different increase rates α are well approximated by those of a homogeneous soil (α = o), fig. 2. The stiffness coefficients k, however, are overestimated by those of an homogeneous soil, which results in a slight underestimation of the equivalent modal damping ratio according to eq. (4).

For low frequencies it can be seen, that the thick layer exhibits no radiation damping. However, in an elastic halfspace the equivalent damping ratio at low frequencies is very small too, especially for rocking and torsion.

The equivalent damping ratio ξ of an inhomogeneous halfspace may be considerably lower than that of an homogeneous halfspace with shear modulus G_o, since $\tilde{a}$ decreases with increasing $\tilde{G}$ and therefore with an increase of α.

A similarly good agreement of the dynamic coefficients for the homogeneous and inhomogeneous soil has been found for the Poisson ratios ν = 0.2 and 0.45. The method has also be applied with satisfying accuracy to rectangular foundations /6/.

CONCLUSIONS

- The dynamic stiffness of a rigid foundation on an inhomogeneous soil in which the shear modulus increases linearly with depth can be expressed by that of a foundation on a homogeneous halfspace. The modulus for an equivalent halfspace corresponds to a representative depth.

- The representative depth for static stiffness is different for vertical, horizontal, rocking and torsional motion.

- For the frequency dependent stiffness and damping coefficients the representative depths are different and depend on the wave length in soil and hence on the frequency of vibration.

- The damping for an inhomogeneous soil may be considerably lower than that for a homogeneous soil which results in the same static foundation stiffness.

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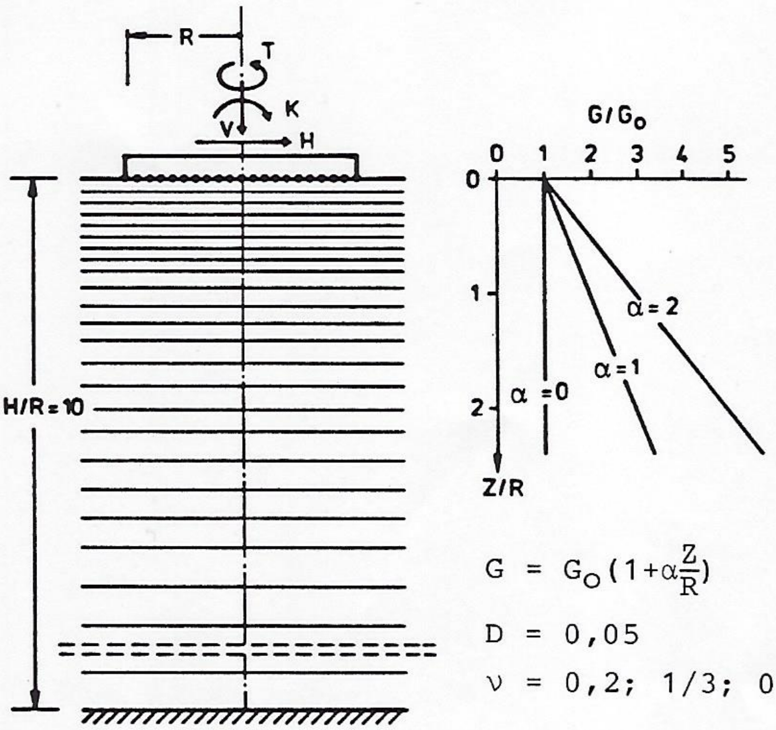
Table I : Representative depth $\zeta = \overline{Z}/R$ for static stiffness

	vertical			horizontal			rocking			torsion
ν	.20	.33	.45	.20	.33	.45	.20	.33	.45	
α = 1	.87	.95	1.08	.50	.50	.50	.36	.40	.46	.20
α = 2	.72	.79	.92	.42	.42	.42	.31	.35	.41	.18
average	.80	.87	1.00	.46			.33	.37	.44	.19



$G = G_O(1+\alpha\frac{Z}{R})$

$D = 0{,}05$

$\nu = 0{,}2;\ 1/3;\ 0{,}45$

Fig. 1 : Soil model

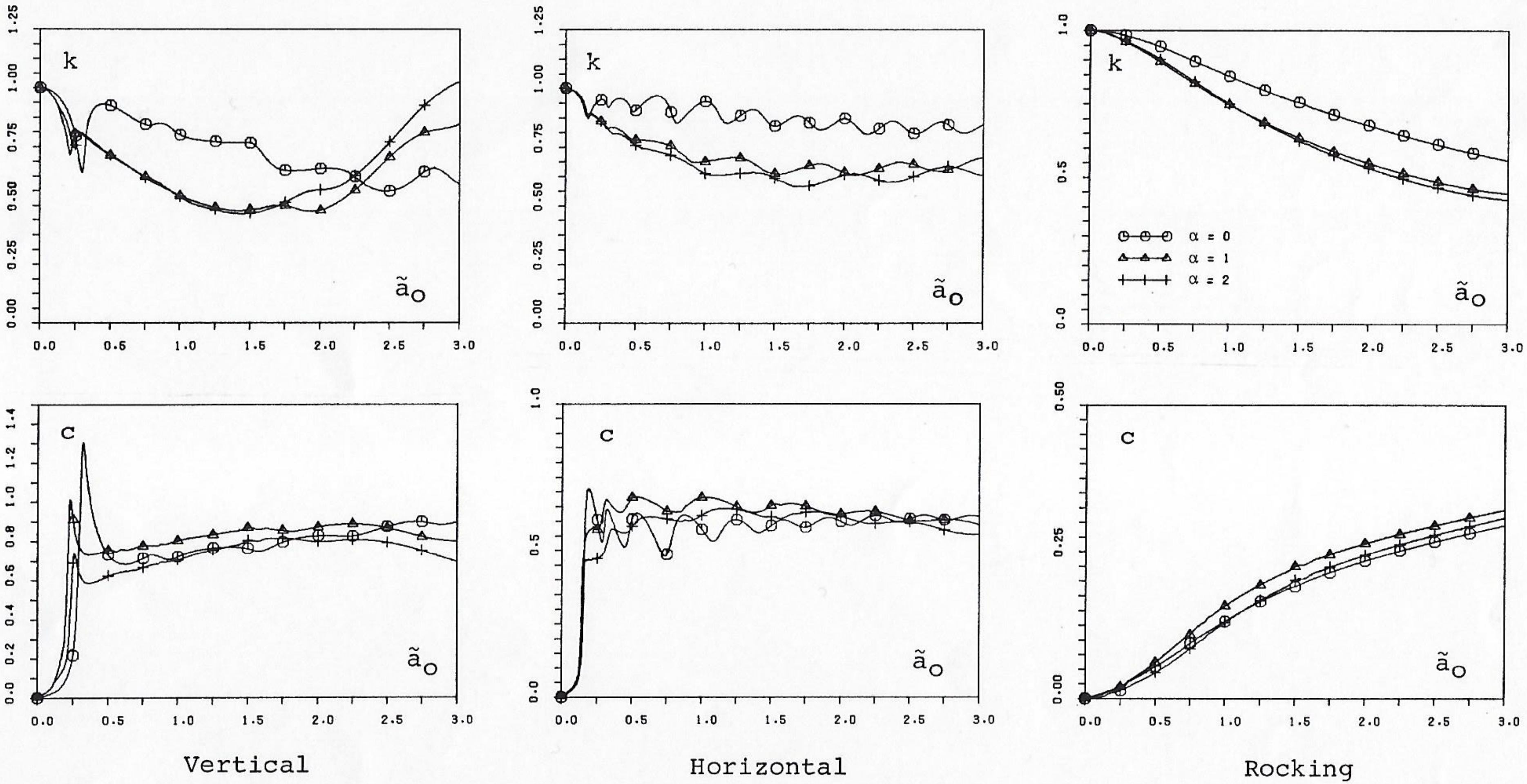
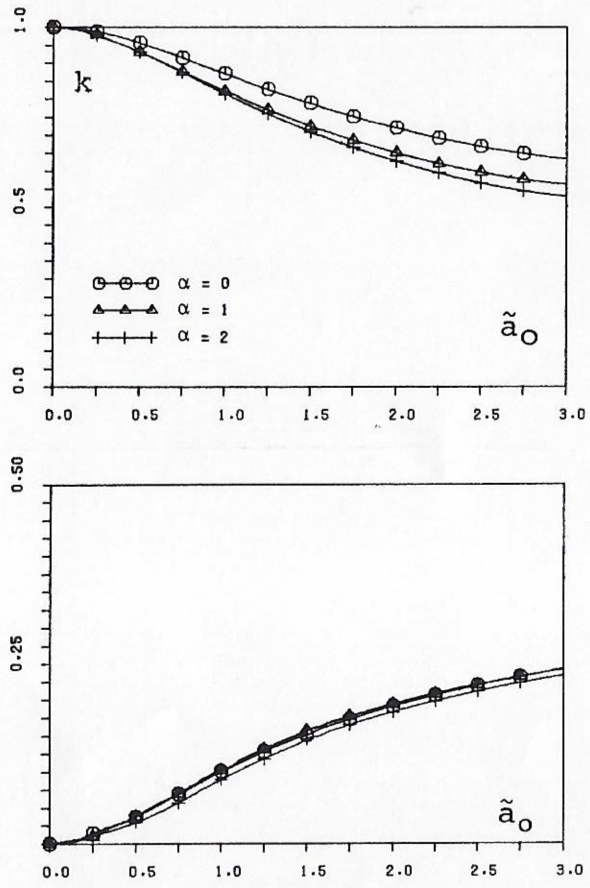


Fig. 2 : Stiffness and damping coefficients for a circular foundation on inhomogeneous soil; $\nu = 1/3$



Torsion

Fig. 2 (cont.)

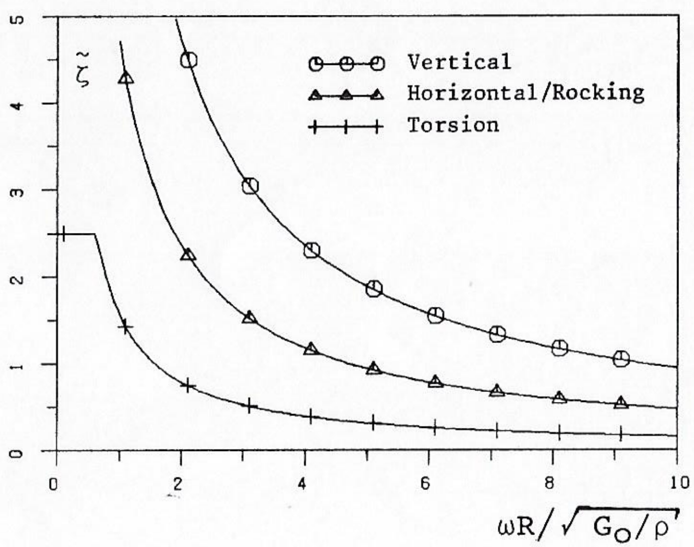


Fig. 3 : Value $\tilde{\zeta} = \tilde{Z}/R$ for dynamic stiffness and damping coefficients