Prof. Dr.-Ing. Horst Werkle 4.5.2022

**RBE3 element -**
**Example 4.16, page327 in "Finite Elements in Structural Analysis" (FIB)**

Denotation and equation numbers of the RBE3 element according to "NX Nastran 12, Advanced Nonlinear Theory and Modeling Guide", Siemens Product Lifecycle Management, 2017, pages 111-115"

Matrices Sk acc. to eq. (2.7-1)

$d := 30$

$$S\_T_1 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{d}{2} & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 15 & 0 & 1 \end{pmatrix} \qquad S_1 := S\_T_1^{T} = \begin{pmatrix} 1 & 0 & 15 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S\_T_2 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad S_2 := S\_T_2^{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$S\_T_3 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-d}{2} & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -15 & 0 & 1 \end{pmatrix} \qquad S_3 := S\_T_3^{T} = \begin{pmatrix} 1 & 0 & -15 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Weighting matrices Wk

$$W_1 := \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{3}{16} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad W_2 := \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{10}{16} & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad W_3 := \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{3}{16} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Matrix X acc. to eq. (2.7-6)

$$X := \left(S\_T_1 \cdot W_1 \cdot S_1 + S\_T_2 \cdot W_2 \cdot S_2 + S\_T_3 \cdot W_3 \cdot S_3\right)^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8.889 \times 10^{-3} \end{pmatrix}$$

**Transmission factors of forces/moments at the reference point**

Matrix Gk acc. to eq. (2.7-6)

$$G_1 := W_1 \cdot S_1 \cdot X = \begin{pmatrix} 0.25 & 0 & 0.033 \\ 0 & 0.188 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \frac{3}{16} = 0.188 \qquad \frac{1}{d} = 0.033$$

$$G_2 := W_2 \cdot S_2 \cdot X = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.625 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \frac{10}{16} = 0.625$$

$$G_3 := W_1 \cdot S_3 \cdot X = \begin{pmatrix} 0.25 & 0 & -0.033 \\ 0 & 0.188 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Composition of submatrices $G_1$, $G_2$, $G_3$:

$$\text{stapeln}\left(G_1, G_2, G_3\right) = \begin{pmatrix} 0.25 & 0 & 0.033 \\ 0 & 0.188 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0.625 & 0 \\ 0 & 0 & 0 \\ 0.25 & 0 & -0.033 \\ 0 & 0.188 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

**The matrix is identical to the matrix in eq. (4.129b), FIB, page 329**

**Weigting factors of displacements**

Matrix in eq. (2.7-10)

$$G_1^{\,T} = \begin{pmatrix} 0.25 & 0 & 0 \\ 0 & 0.188 & 0 \\ 0.033 & 0 & 0 \end{pmatrix} \qquad G_2^{\,T} = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.625 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad G_3^{\,T} = \begin{pmatrix} 0.25 & 0 & 0 \\ 0 & 0.188 & 0 \\ -0.033 & 0 & 0 \end{pmatrix}$$

Composition of submatrices

$$\text{erweitern}\left(G_1^{\,T}, G_2^{\,T}, G_3^{\,T}\right) = \begin{pmatrix} 0.25 & 0 & 0 & 0.5 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0.188 & 0 & 0 & 0.625 & 0 & 0 & 0.188 & 0 \\ 0.033 & 0 & 0 & 0 & 0 & 0 & -0.033 & 0 & 0 \end{pmatrix}$$

**The matrix is identical with the matix in eq. (4.129a), FIB, page 328**