Finite Elements in Structural Analysis

1 Introduction

Truss and beam structures

Plate and shell structures

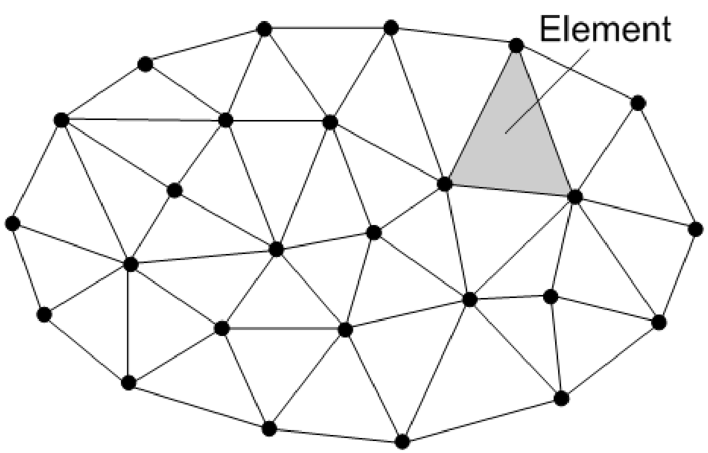
Modeling

Definition

The Finite Element Method (FEM) is a mathematical method for the numerical solution of differential equations. Physical problems can be described mathematically by differential equations in an idealized form.

Basic idea of the FEM:

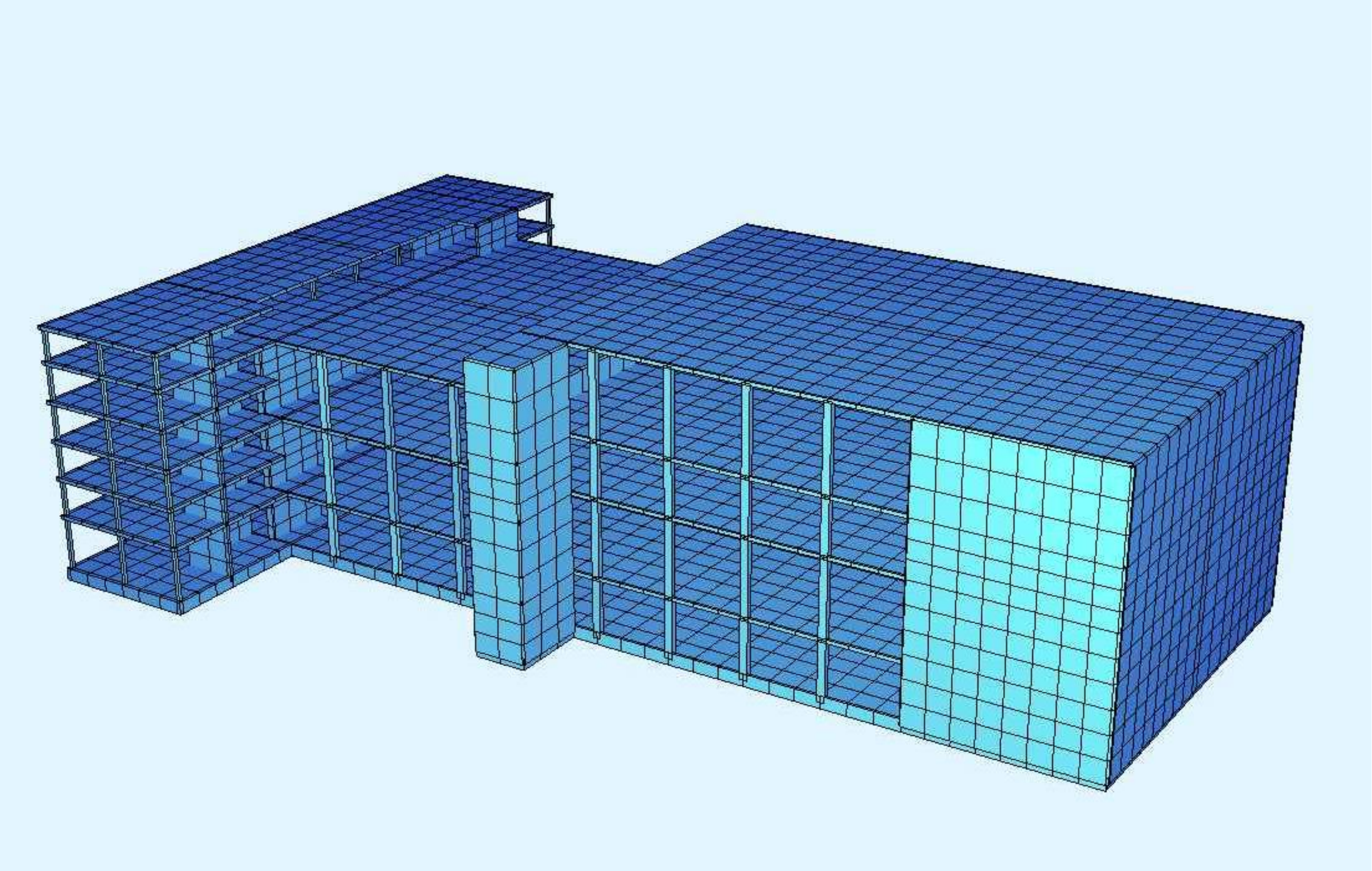
The global behaviour of complex structures can be approximated through simple interpolation functions with unknown parameters for sections (elements).



Computer method (thousends of unknowns)

FEA in civil engineering

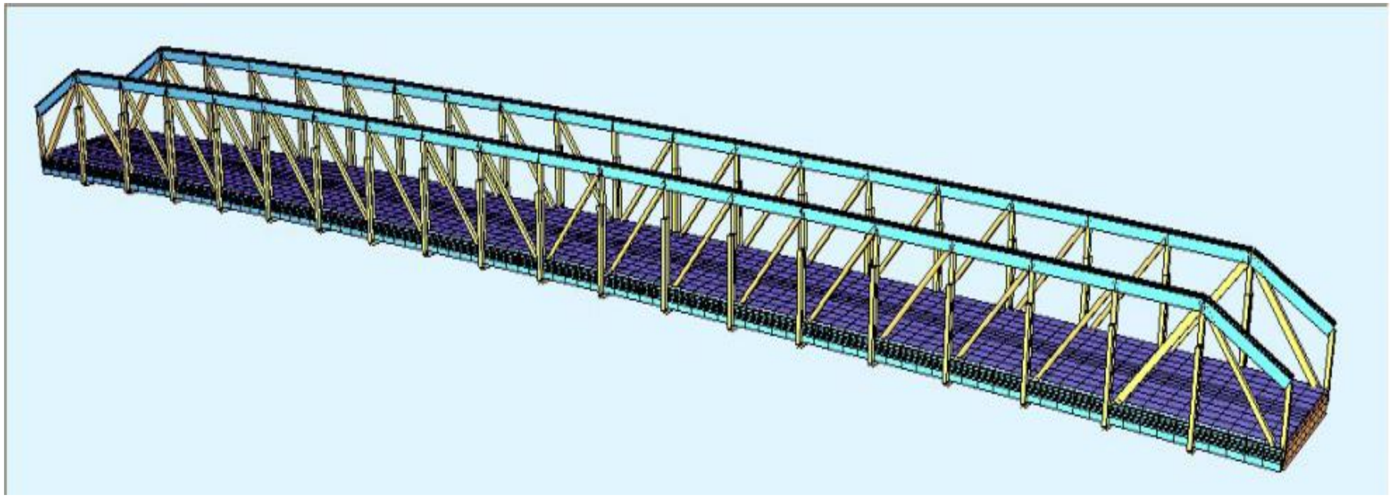
Building



FEM software
Sofistik /Germany

FEA in civil engineering

Bridge

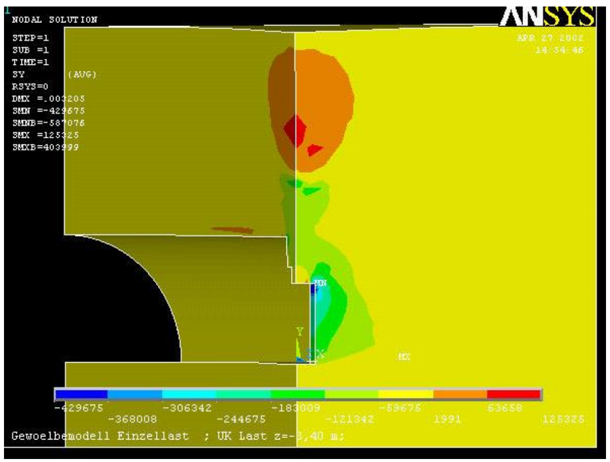


FEM software
Sofistik /Germany

FEA in civil engineering

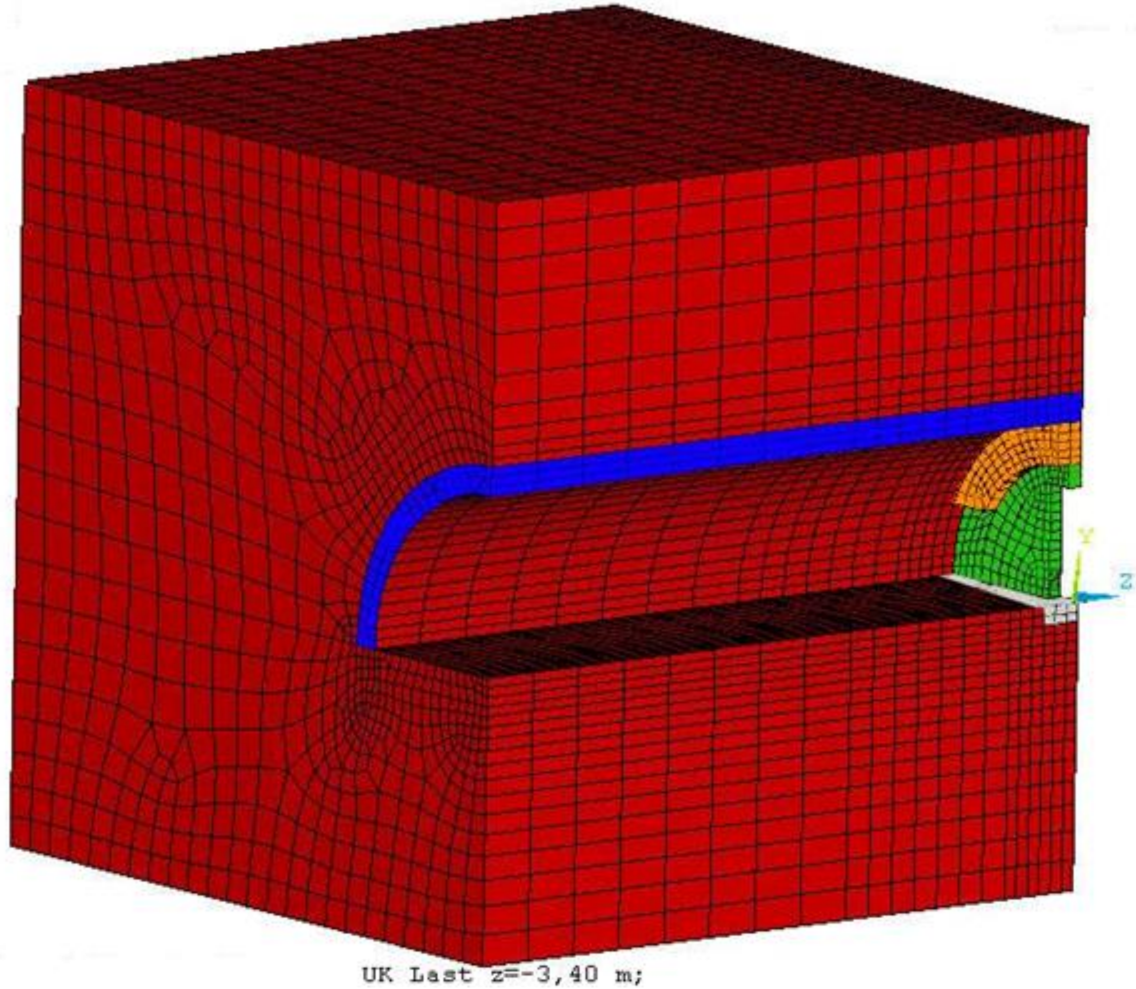
Analysis of a historical tunnel

FEM software
ANSYS



Vertical stresses

3D-Finite element model

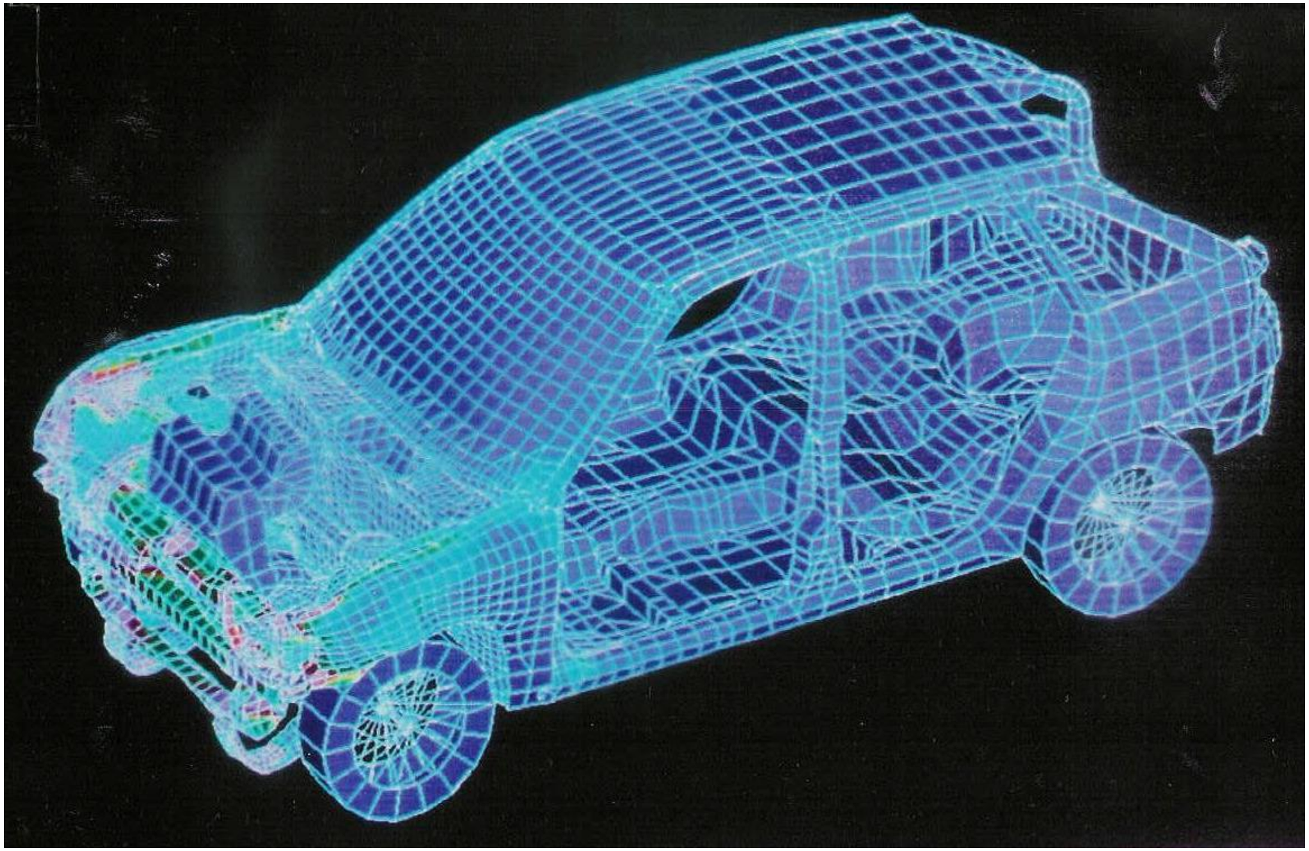


FEA in automotive engineering

Simulation of crash tests

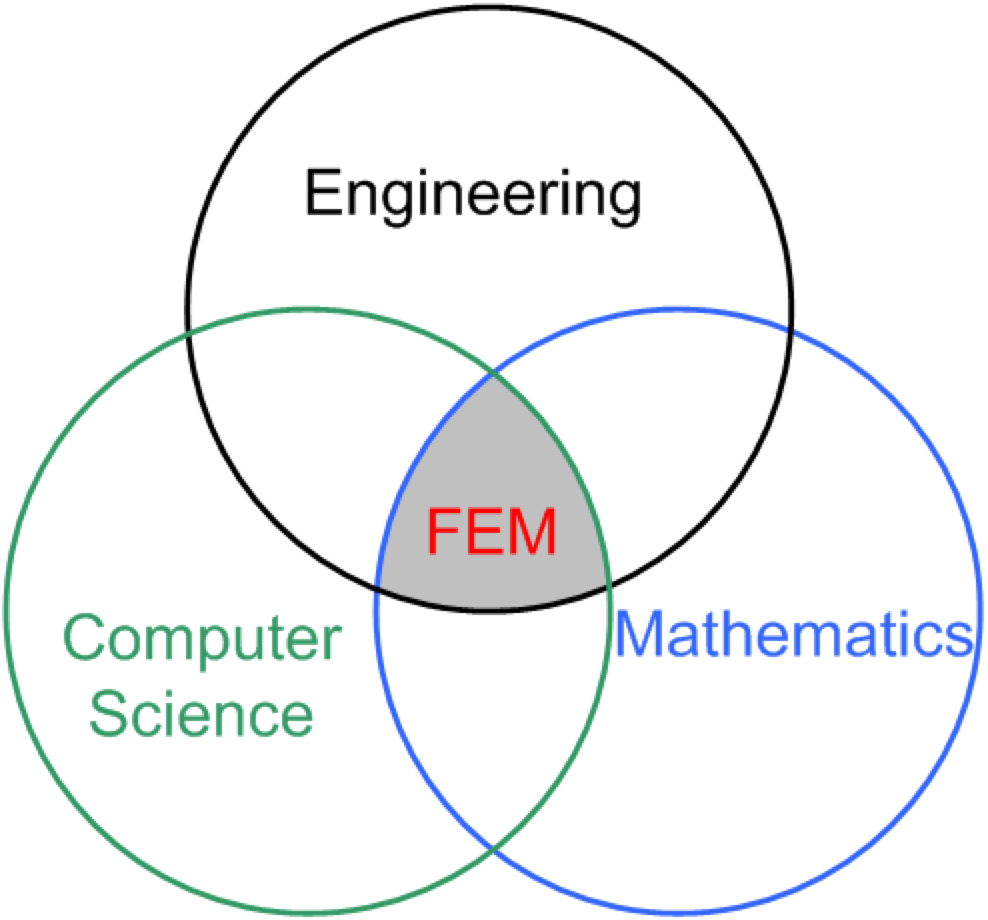
Body of a car

Opel AG, Rüsselsheim



FE analysis

- **complex geometry**
- **nonlinear material**
- **geometrically nonlinear behavior**
- **dynamics**


Engineering
FEM
Computer
Science
Mathematics

Precursors in mathematics

- **Ritz W.**
  Über eine neue Methode zur Lösung gewisser Variationsprobleme der mathematischen Physik, 1909,
  (in German)

Walther Ritz

- **Galerkin, B. G.**
  Series solution of some problems of elastic equilibrium of rods and plates, 1915
  (in Russian)

Boris Galerkin

The 1950s

- Matrix theory of structural analysis (Argyris 1954, 1956)
- First paper on the Finite Element Method:

Argyris

Stiffness and deflection analysis of complex structures,
Turner, Clough, Martin und Topp, 1956

Clough

The 1960s

- First mention of the name **Finite Element**
  Clough, The Finite Element in Plane Stress Analysis, 2nd A.S.C.E. Conference on Electronic Computation, Pittsburgh, 1960
- Fundamental development of the FEM
- 3-D structures, non-linear computations, heat transfer, fluid dynamics, electromagnetism, time-dependent problems
- Development of alternative approaches:
  - Hybrid elements (Pian, 1964)
  - Boundary element method (Rizzo, 1967)

Nastran

Programs

Classical books

The 1970s

- Development of new element types and analysis, also in the non-linear area
- FEM is understood as a mathematical method for the solution of partial differential equations (error estimation, convergence behaviour)
- Development of special finite elements
  - Cyclic symmetric systems
  - Elements for infinite systems (Transmitting Boundaries)

Hardware

The 1980s / 1990s

- New element types for plates and plates in plane stress
- Mathematical accuracy of the FEM: error estimation
- Methods for adaptive mesh refinement
- Porting of mainframe computer software to PC´s and workstations
- Integration of the FEM in CAD work environment

The 2000s

- Nonlinear methods
- Optimization of components and structures
- New numerical methods: meshfree methods, isogeometric analysis
- CFD (Computational fluid dynamics)
- Multiphysics

An early warning

SAP – Structural Analysis Program

by E. Wison, Berkeley, California

The „SAP" warning

The slang name SAP* was selected
to remind the user that this program,
like all computer programs,
lacks intelligence.
It is the responsibility of the engineer
to idealize the structure correctly
and assume responsability for the results.

Edward L. Wilson, Foreword of the first SAP manual, 1970

*) Macmillan dictionary: „*Someone who trusts people too much and can easily be cheated*"

End

1 Introduction

Truss and beam structures

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First paper on the Finite Element Method, 1956

Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP**

Received June 29, 1955. This paper is based on a paper presented at the Aeroelasticity Session, Twenty-Second Annual Meeting, IAS, New York, January 25–29, 1954.

* Structural Dynamics Unit Chief, Boeing Airplane Company, Seattle Division.

† Associate Professor of Civil Engineering, University of California, Berkeley.

‡ Professor of Aeronautical Engineering, University of Washington, Seattle.

** Structures Engineer, Structural Dynamics Unit, Boeing Airplane Company, Wichita Division.

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Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP**

ABSTRACT

A method is developed for calculating stiffness influence coefficients of complex shell-type structures. The object is to provide a method that will yield structural data of sufficient accuracy to be adequate for subsequent dynamic and aeroelastic analyses.

Stiffness of the complete structure is obtained by summing stiffnesses of individual units. Stiffnesses of typical structural components are derived in the paper. Basic conditions of continuity and equilibrium are established at selected points (nodes) in the structure. Increasing the number of nodes increases the accuracy of results. Any physically possible support conditions can be taken into account. Details in setting up the analysis can be performed by nonengineering trained personnel; calculations are conveniently carried out on automatic digital computing equipment.

Method is illustrated by application to a simple truss, a flat plate, and a box beam. Due to shear lag and spar web deflection, the box beam has a 25 per cent greater deflection than predicted from beam theory. It is shown that the proposed method correctly accounts for these effects.

Considerable extension of the material presented in the paper is possible.

(I) INTRODUCTION

PRESENT CONFIGURATION TRENDS in the design of high-speed aircraft have created a number of difficult, fundamental structural problems for the worker in aeroelasticity and structural dynamics. The chief problem in this category is to predict, for a given elastic structure, a comprehensive set of load-deflection relations which can serve as structural basis for dynamic load calculations, theoretical vibration and flutter analyses, estimation of the effects of structural deflection on static air loads, and theoretical analysis of aeroelastic effects on stability and control. This is a problem of exceptional difficulty when thin wings and tail surfaces of low aspect ratio, either swept or unswept, are involved.

It is recognized that camber bending (or rib bending) is a significant feature of the vibration modes of the newer configurations, even of the low-order modes; in order to encompass these characteristics it seems likely that the load-deflection relations of a practical structure must be expressed in the form of either deflection or stiffness influence coefficients. One approach is to employ structural models and to determine the influence coefficients experimentally; it is anticipated that the experimental method will be employed extensively in the future, either in lieu of or as a final check on the result of analysis. However, elaborate models are expensive, they take a long time to build, and tend to become obsolete because of design changes; for these reasons it is considered essential that a continuing research effort should be applied to the development of analytical methods. It is to be expected that modern developments in high-speed digital computing machines will make possible a more fundamental approach to the problems of structural analysis; we shall expect to base our analysis on a more realistic and detailed conceptual model of the real structure than has been used in the past. As indicated by the title, the present paper is exclusively concerned with methods of theoretical analysis; also it is our object to outline the development of a method that is well adapted to the use of high-speed digital computing machinery.

(II) REVIEW OF EXISTING METHODS OF STRUCTURAL ANALYSIS

(1) *Elementary Theories of Flexure and Torsion*

The limitations of these venerable theories are too well known to justify extensive comment. They a

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805

Abstract – Turner, Clough, Martin, Topp, 1956

Stiffness of the complete structure is obtained by summing stiffnesses of individual units. Stiffnesses of typical structural components are derived in the paper. Basic conditions of continuity and equilibrium are established at selected points (nodes) in the structure. Increasing the number of nodes increases the accuracy of results. Any physically possible support conditions can be taken into account. Details in setting up the analysis can be performed by nonengineering trained personnel; calculations are conveniently carried out on automatic digital computing equipment.

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Eine neue Statik – Statik in Matrizenformulierung

Wir sind uns seit einigen Jahren bewusst, dass keine der gewöhnlichen statischen Methoden wirklich geeignet ist, die Spannungsverteilung und die Nachgiebigkeits-matrizen der hochgradig statisch unbestimmten Systeme der modernen Luftfahrt-konstruktionen zu bestimmen. Ähnliche Schwierigkeiten treten auch in anderen Anwendungsgebieten der Statik auf. Die Iterationsverfahren können in gewissen Fällen nützlich sein, sind aber im allgemeinen zu langwierig und haben sich nicht bei den membran- und schalenförmigen Tragwerken der Luftfahrt bewährt.

Diese Schwierigkeiten können wir mit der Matrizenformulierung der Statik in Verbindung mit dem elektronischen Digitalautomaten überwinden. Die Matrizenformulierung erlaubt nicht nur, die Rechnungen viel übersichtlicher zu gestalten, sondern ist auch die ideale Schreibweise für den Digitalautomaten. Außerdem sind die theoretischen Ableitungen der Matrizentheorie so durchsichtig und elegant, dass neue und praktisch wertvolle Beziehungen, die in der gewöhnlichen Schreibweise unmöglich oder nur schwierig erkennbar wären, sich jetzt sehr einfach ergeben.

J.H. Argyris, Die Matrizentheorie der Statik, Ingenieur-Archiv, 1956

Classical books

- **Pestel/Leckie**
  Matrix Methods in Elastomechanics, 1964
- **Livesley, R.H.**
  Matrix Methods of Structural Analysis, 1964
- **Przemieniecki**
  Theory of Matrix Structural Analysis, 1968
- **Zienkiewicz**
  The Finite Element Method in Engineering Science, 1971
- **Gallagher**
  Finite Element Analysis, 1975
- **Bathe, Wilson**
  Numerical Methods in Finite Element Analysis, 1976

Development of FE programs (1965 – 1975)

STRUDL (STRUctural Design Language), MIT, USA, 1962-64, 65-71
- GTSTRUDL

NASTRAN (NASA Structural Analysis Program), USA, 1965-1972
- MSC-Nastran
- CSA-Nastran

ASKA (1970) Argyris, Stuttgart

ANSYS Swanson Analysis Systems, 1970

STARDYNE Mechanical Research, Rosen&Ragle, USA, 1966-1967

SAP (Structural Analysis Program), Wilson, Bathe, Berkeley, USA, 1970
- SAPIV / SAP80 / SAP90 / SAP2000

ADINA (Bathe, MIT Boston, USA)