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# Finite Elemente in der Baustatik

## 1 Einführung

Stabtragwerke

Flächentragwerke

Modellbildung

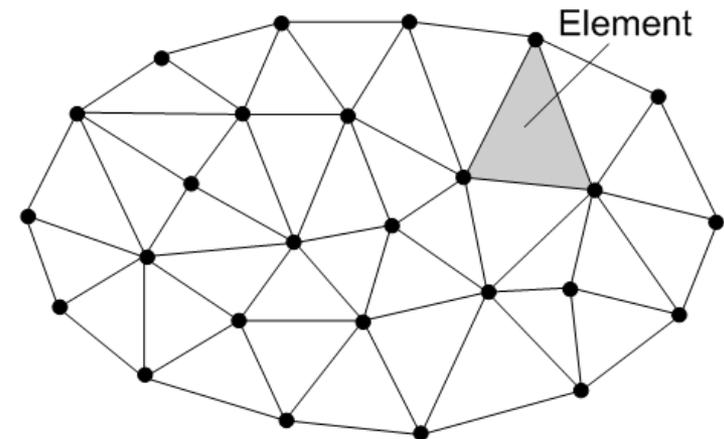
## Definition

Die Finite Element Methode (FEM) ist eine mathematische Methode zur numerischen **Lösung von Differentialgleichungen**. **Physikalische Probleme** können in in idealisierter Form durch **Differentialgleichungen** beschrieben werden.

### Grundlegende Idee der FEM:

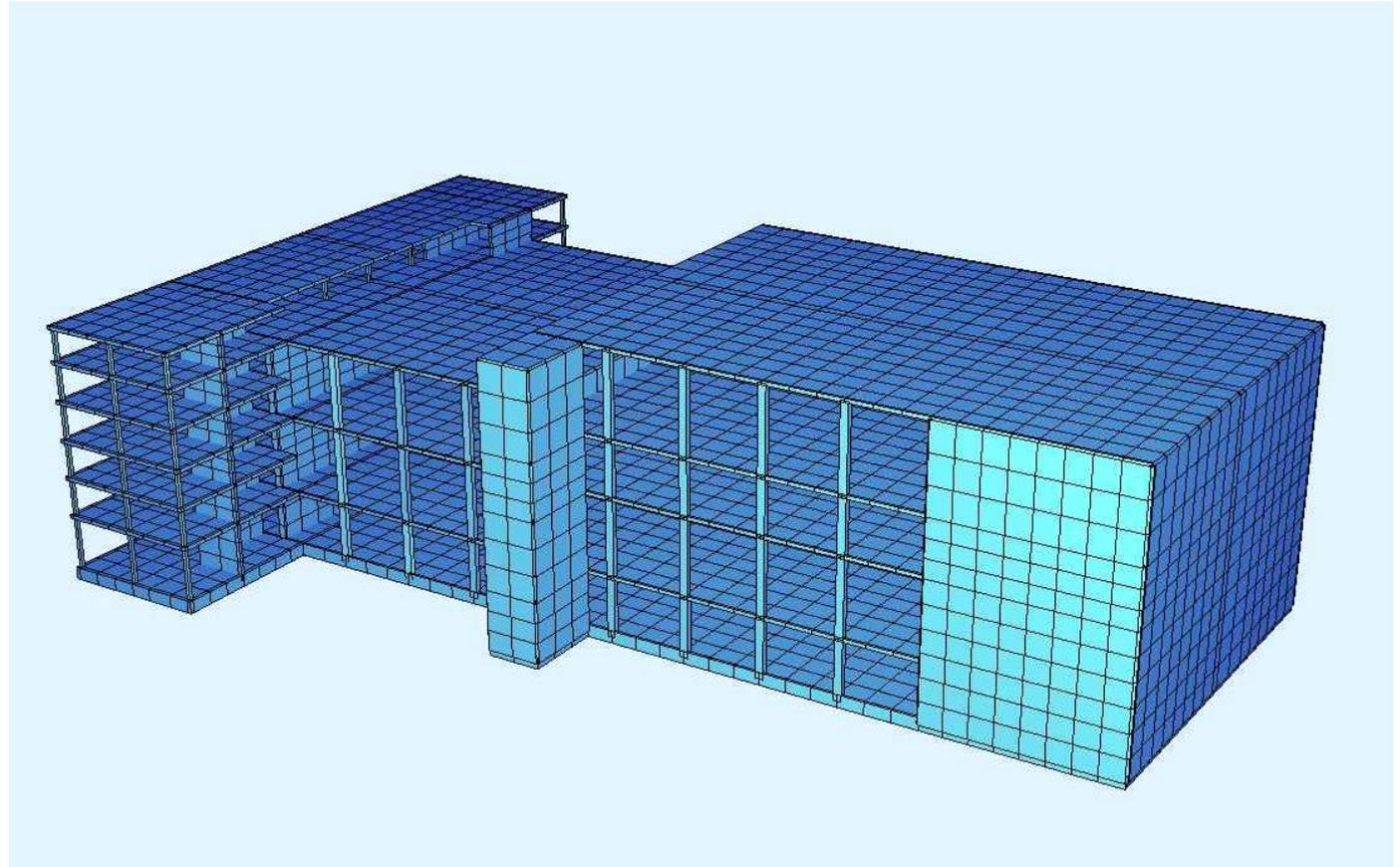
Das Verhalten komplexer Strukturen kann durch einfache Interpolationsfunktionen angenähert werden. Unbekannte sind die Parameter an den Verbindungspunkten der Abschnitte (Elemente).

**Computerverfahren** (tausende Unbekannte)



## FEM im Bauingenieurwesen

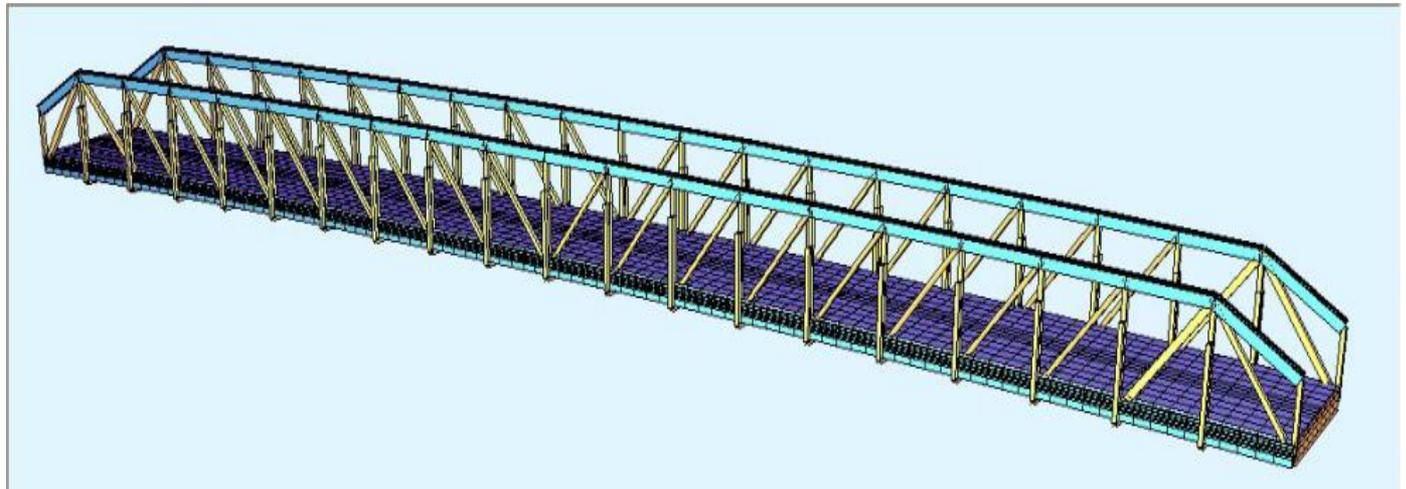
Bauwerk



FEM software  
Sofistik /Germany

## FEM im Bauingenieurwesen

### Brücke

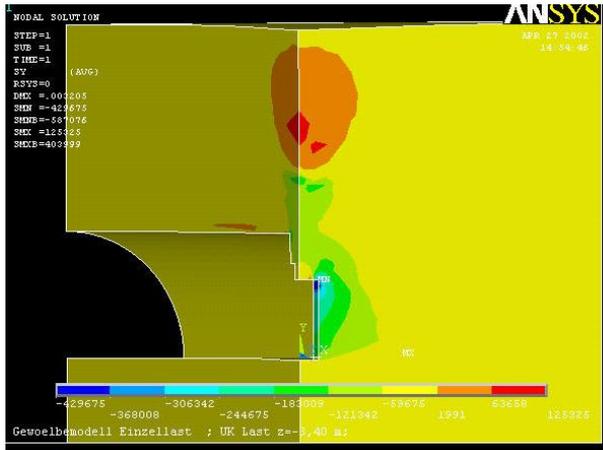


FEM software  
Sofistik /Germany

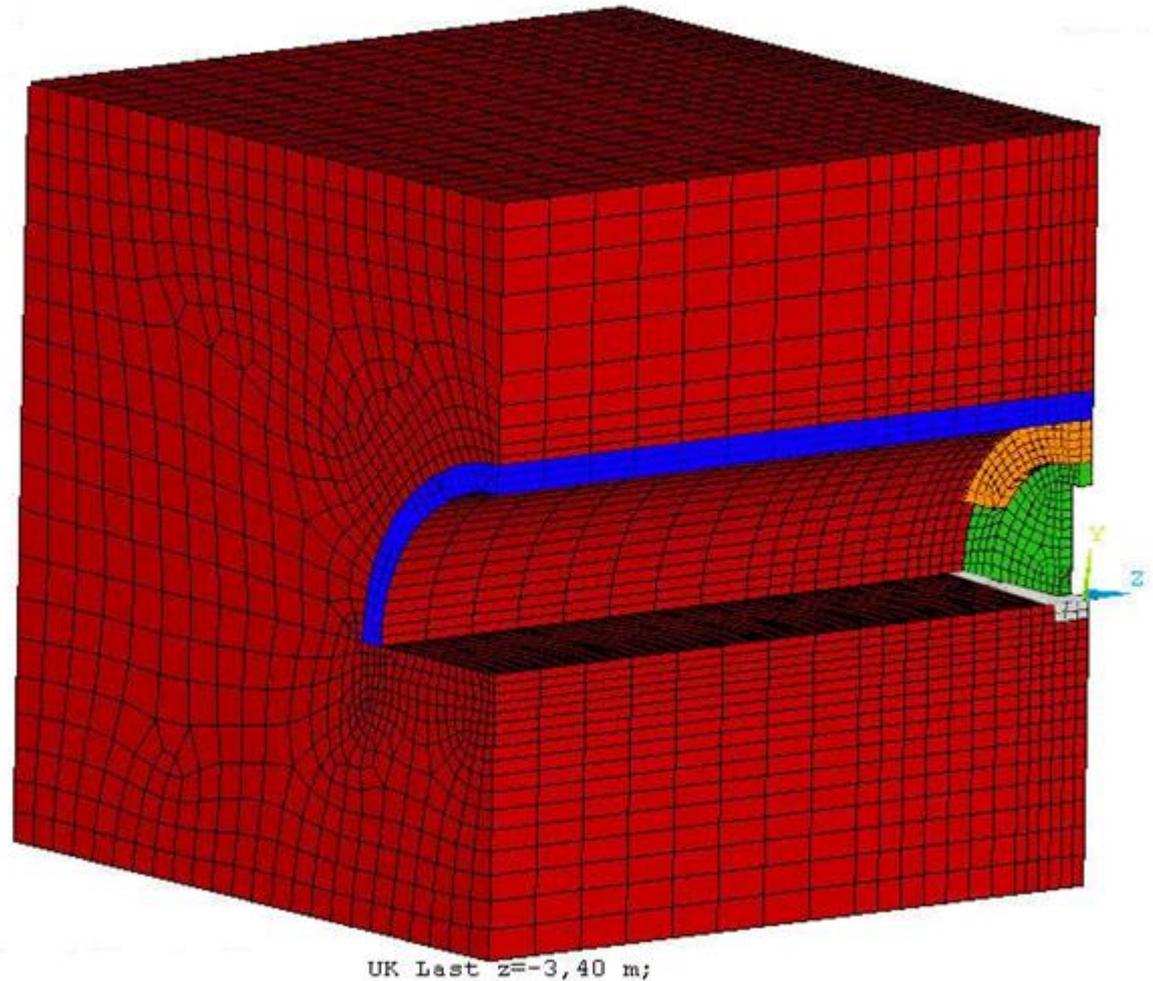
# FEM im Bauingenieurwesen

## Berechnung eines historischen Tunnels

FEM software  
ANSYS



Vertikalspannungen



3D-Finite-Element-Modell

## FEM im Automobilbau

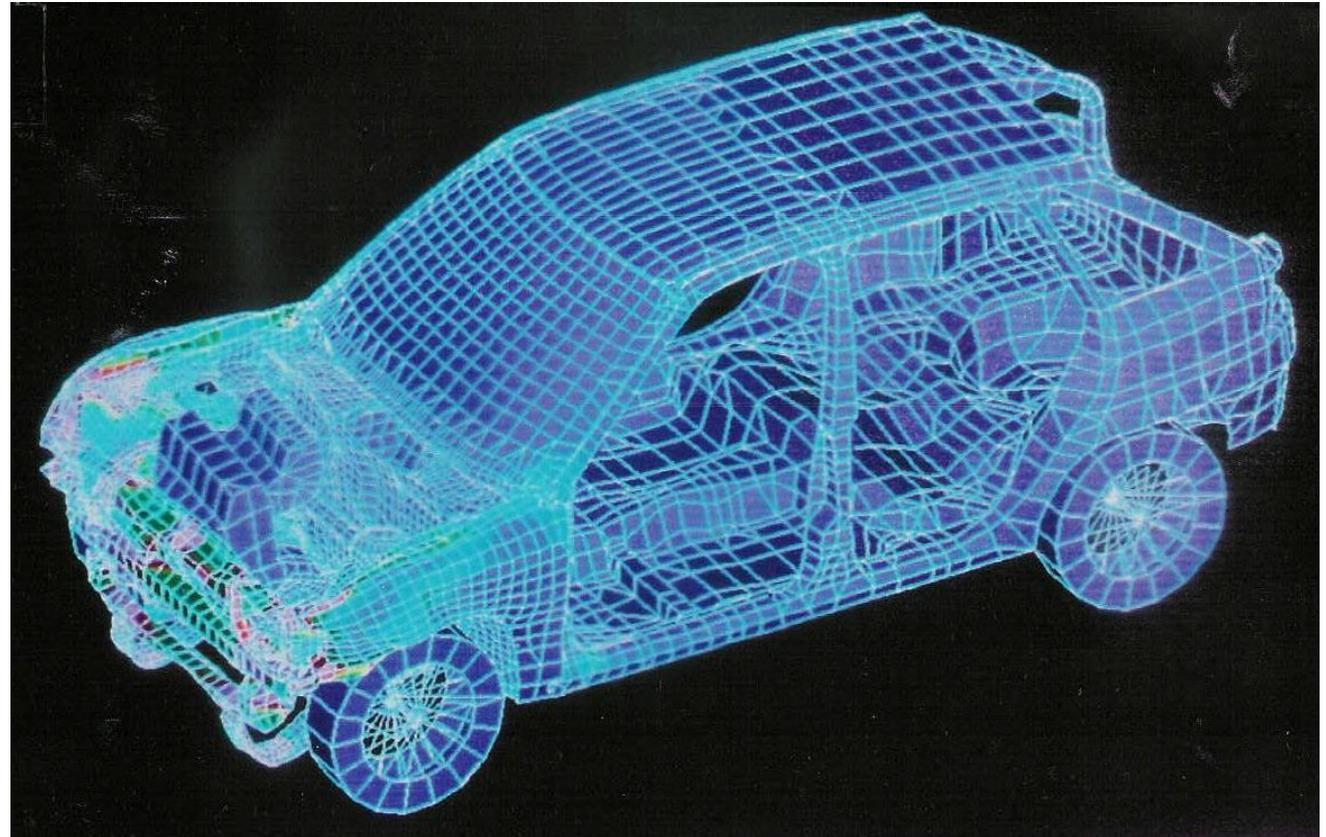
### Simulation von crash Tests

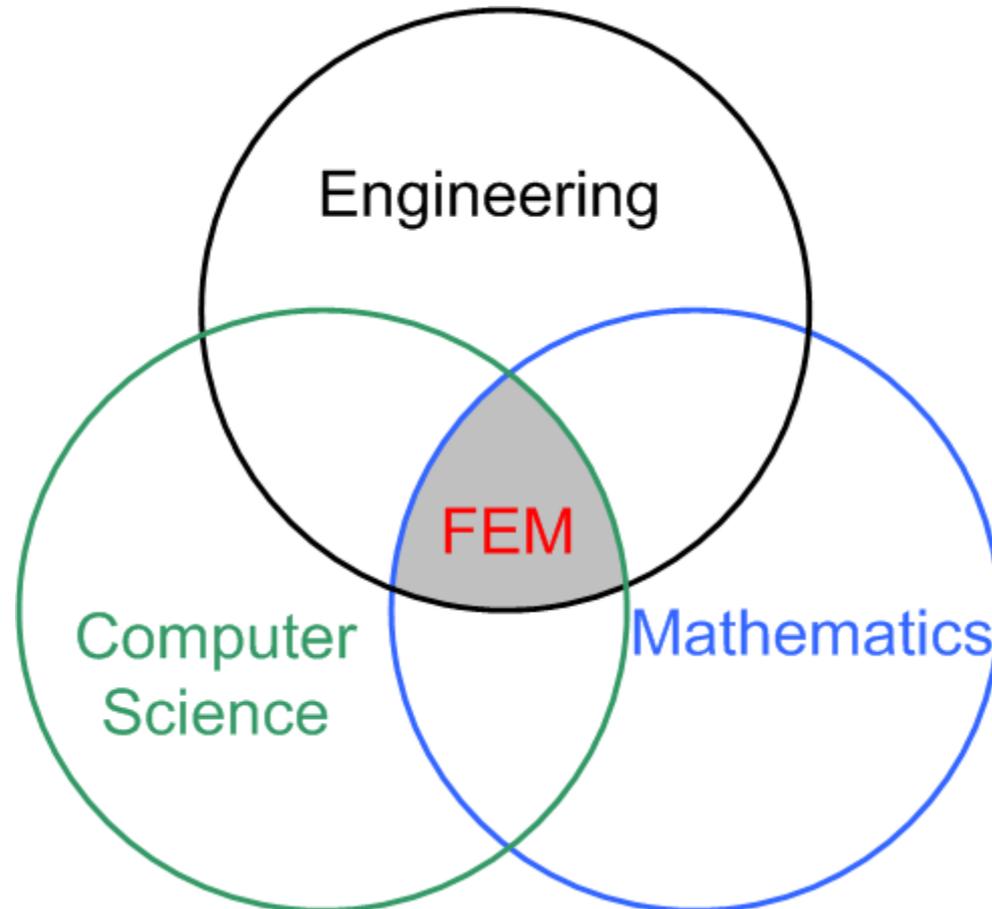
#### Fahrzeugkarserie

Opel AG, Rüsselsheim

#### FE Berechnung

- komplexe Geometrie
- Nichtlineares Materialverhalten
- geometrisch nichtlineares Verhalten
- Dynamik

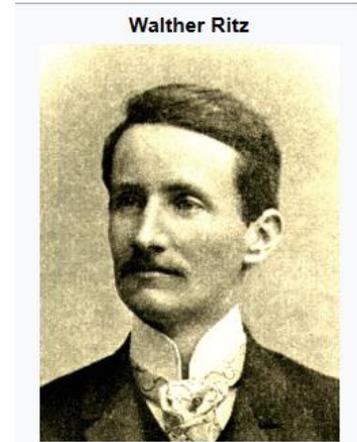




## Vorläufer in der Mathematik

➤ **Ritz W.**

*Über eine neue Methode zur Lösung gewisser Variationsprobleme der mathematischen Physik, 1909,  
(in German)*



➤ **Galerkin, B. G.**

*Series solution of some problems of elastic equilibrium of rods and plates, 1915  
(in Russian)*



## 1950er Jahre

- Matrizenmethode in der Baustatik (Argyris 1954, 1956)
- Erste Veröffentlichung zur Finite Elemente Methode:

Argyris

***Stiffness and deflection analysis of complex structures,  
Turner, Clough, Martin und Topp, 1956***

Clough

## 1960er Jahre

- Erstmalige Bezeichnung als **Finite Elemente** in *Clough, The Finite Element in Plane Stress Analysis, 2nd A.S.C.E. Conference on Electronic Computation, Pittsburgh, 1960*
- Entwicklung der Grundlagen der FEM
- Dreidimensionale Strukturen, nichtlineare Berechnungen, Wärmeströmungen, Fluid Dynamik, Elektromagnetismus, zeitabhängige Probleme
- Entwicklung alternativer Ansätze:
  - Hybride Elemente (Pian, 1964)
  - Randelementmethode (Rizzo, 1967)

Nastran

Programme

FEM-Bücher

## 1970er Jahre

- Entwicklung neuer Elementtypen, nichtlineare Elemente
- Die FEM wird als mathematische Methode zur Lösung partieller Differentialgleichungen verstanden (mit Fehlerabschätzungen, Nachweis des Konvergenzverhaltens u.s.w.).
- Entwicklung spezieller Finite Elemente
  - Elemente für Systeme mit zyklischer Symmetrie
  - Elemente für unendlich ausgedehnte Systeme (Transmitting Boundaries)

## 1980er und 1990er Jahre

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- Neue Elementtypen für Platten und Schalen
- Mathematische Genauigkeit der FEM: Fehlerabschätzungen
- Methoden zur adaptiven Netzverfeinerung
- Portierung von FE-Software von Großcomputern zu PC's und Workstations
- Integration der FEM in eine CAD-Arbeitsumgebung

## 2000er Jahre

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- Nichtlineare Methoden
- Optimierung von Bauteilen und Tragwerken
- Neue numerische Methoden: Netzfremde Methoden, Isogeometrische Analyse
- CFD (Computational fluid dynamics)
- Multiphysics

## Warnung im Handbuch eines der ersten FE-Programme

### **SAP** – **S**tructural **A**nalysis **P**rogram

by E. Wilson, Berkeley, California

## The „SAP“ warning

The slang name SAP\* was selected to remind the user that this program, like all computer programs, lacks intelligence.

It is the responsibility of the engineer to idealize the structure correctly and assume responsibility for the results.

Edward L. Wilson, Foreword of the first SAP manual, 1970

\*) Macmillan dictionary: „*Someone who trusts people too much and can easily be cheated*“

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# Ende

## 1 Einführung

Stabwerke

Flächentragwerke

Modellbildung

# Erste Veröffentlichung zur Finite-Elemente-Methode, 1956

## Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,\* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP\*\*

Received June 29, 1955. This paper is based on a paper presented at the Aeroelasticity Session, Twenty-Second Annual Meeting, IAS, New York, January 25-29, 1954.

\* Structural Dynamics Unit Chief, Boeing Airplane Company, Seattle Division.

† Associate Professor of Civil Engineering, University of California, Berkeley.

‡ Professor of Aeronautical Engineering, University of Washington, Seattle.

\*\* Structures Engineer, Structural Dynamics Unit, Boeing Airplane Company, Wichita Division.

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VOLUME 23

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## Stiffness and Deflection Analysis of Complex Structures

M. J. TURNER,\* R. W. CLOUGH,† H. C. MARTIN,‡ AND L. J. TOPP\*\*

### ABSTRACT

A method is developed for calculating stiffness influence coefficients of complex shell-type structures. The object is to provide a method that will yield structural data of sufficient accuracy to be adequate for subsequent dynamic and aeroelastic analyses. Stiffness of the complete structure is obtained by summing stiffnesses of individual units. Stiffnesses of typical structural components are derived in the paper. Basic conditions of continuity and equilibrium are established at selected points (nodes) in the structure. Increasing the number of nodes increases the accuracy of results. Any physically possible support conditions can be taken into account. Details in setting up the analysis can be performed by nonengineering trained personnel; calculations are conveniently carried out on automatic digital computing equipment.

Method is illustrated by application to a simple truss, a flat plate, and a box beam. Due to shear lag and spar web deflection, the box beam has a 25 per cent greater deflection than predicted from beam theory. It is shown that the proposed method correctly accounts for these effects.

Considerable extension of the material presented in the paper is possible.

### (I) INTRODUCTION

PRESENT CONFIGURATION TRENDS in the design of high-speed aircraft have created a number of difficult, fundamental structural problems for the worker in aeroelasticity and structural dynamics. The chief problem in this category is to predict, for a given elastic structure, a comprehensive set of load-deflection relations which can serve as structural basis for dynamic load calculations, theoretical vibration and flutter analyses, estimation of the effects of structural deflec-

tion on static air loads, and theoretical analysis of aeroelastic effects on stability and control. This is a problem of exceptional difficulty when thin wings and tail surfaces of low aspect ratio, either swept or unswept, are involved.

It is recognized that camber bending (or rib bending) is a significant feature of the vibration modes of the newer configurations, even of the low-order modes; in order to encompass these characteristics it seems likely that the load-deflection relations of a practical structure must be expressed in the form of either deflection or stiffness influence coefficients. One approach is to employ structural models and to determine the influence coefficients experimentally; it is anticipated that the experimental method will be employed extensively in the future, either in lieu of or as a final check on the result of analysis. However, elaborate models are expensive, they take a long time to build, and tend to become obsolete because of design changes; for these reasons it is considered essential that a continuing research effort should be applied to the development of analytical methods. It is to be expected that modern developments in high-speed digital computing machines will make possible a more fundamental approach to the problems of structural analysis; we shall expect to base our analysis on a more realistic and detailed conceptual model of the real structure than has been used in the past. As indicated by the title, the present paper is exclusively concerned with methods of theoretical analysis; also it is our object to outline the development of a method that is well adapted to the use of high-speed digital computing machinery.

### (II) REVIEW OF EXISTING METHODS OF STRUCTURAL ANALYSIS

#### (1) Elementary Theories of Flexure and Torsion

The limitations of these venerable theories are too well known to justify extensive comment. They a

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## Abstract – Turner, Clough, Martin, Topp, 1956

Stiffness of the complete structure is obtained by summing stiffnesses of individual units. Stiffnesses of typical structural components are derived in the paper. Basic conditions of continuity and equilibrium are established at selected points (nodes) in the structure. Increasing the number of nodes increases the accuracy of results. Any physically possible support conditions can be taken into account. Details in setting up the analysis can be performed by nonengineering trained personnel; calculations are conveniently carried out on automatic digital computing equipment.

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## Eine neue Statik – Statik in Matrizenformulierung

Wir sind uns seit einigen Jahren bewusst, dass keine der gewöhnlichen statischen Methoden wirklich geeignet ist, die Spannungsverteilung und die Nachgiebigkeitsmatrizen der hochgradig statisch unbestimmten Systeme der modernen Luftfahrtkonstruktionen zu bestimmen. Ähnliche Schwierigkeiten treten auch in anderen Anwendungsgebieten der Statik auf. Die Iterationsverfahren können in gewissen Fällen nützlich sein, sind aber im allgemeinen zu langwierig und haben sich nicht bei den membran- und schalenförmigen Tragwerken der Luftfahrt bewährt.

Diese Schwierigkeiten können wir mit der Matrizenformulierung der Statik in Verbindung mit dem elektronischen Digitalautomaten überwinden. Die Matrizenformulierung erlaubt nicht nur, die Rechnungen viel übersichtlicher zu gestalten, sondern ist auch die ideale Schreibweise für den Digitalautomaten. Außerdem sind die theoretischen Ableitungen der Matrizenformulierung so durchsichtig und elegant, dass neue und praktisch wertvolle Beziehungen, die in der gewöhnlichen Schreibweise unmöglich oder nur schwierig erkennbar wären, sich jetzt sehr einfach ergeben.

*J.H. Argyris, Die Matrizenformulierung der Statik, Ingenieur-Archiv, 1956*



## Klassische FE-Bücher

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- **Pestel/Leckie**  
*Matrix Methods in Elastomechanics, 1964*
- **Livesley, R.H.**  
*Matrix Methods of Structural Analysis, 1964*
- **Przemieniecki**  
*Theory of Matrix Structural Analysis, 1968*
- **Zienkiewicz**  
*The Finite Element Method in Engineering Science, 1971*
- **Gallagher**  
*Finite Element Analysis, 1975*
- **Bathe, Wilson**  
*Numerical Methods in Finite Element Analysis, 1976*



## Entwicklung von FE-Programmen (1965 – 1975)

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- STRUDL** (STRUctural Design Language), MIT, USA, 1962-64, 65-71
- GTSTRUDL
- NASTRAN** (NASA Structural Analysis Program), USA, 1965-1972
- MSC-Nastran
  - CSA-Nastran
- ASKA** (1970) Argyris, Stuttgart
- ANSYS** Swanson Analysis Systems, 1970
- STARDYNE** Mechanical Research, Rosen&Ragle, USA, 1966-1967
- SAP** (Structural Analysis Program), Wilson, Bathe, Berkeley, USA, 1970
- SAPIV / SAP80 / SAP90 / SAP2000
- ADINA** (Bathe, MIT Boston, USA)

