Finite Elements in Structural Analysis

Introduction

Truss and beam structures

3 Plate, shell and solid structures

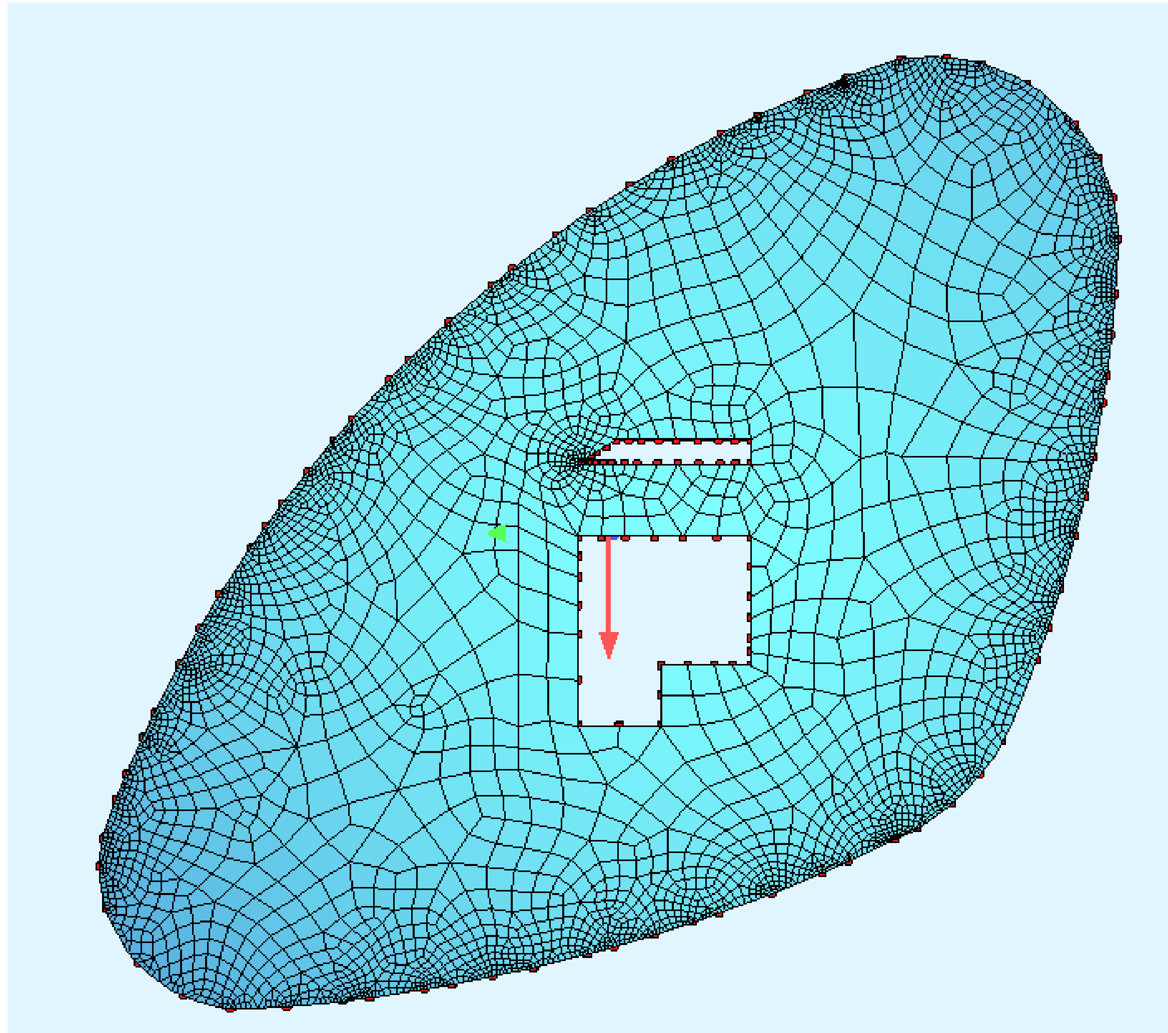
Modeling

Example: Structural slab with openings

New building of the

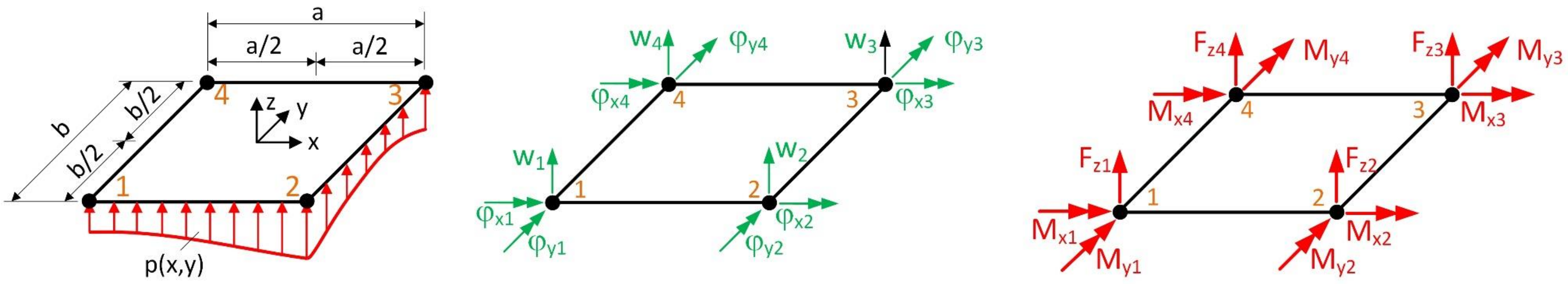
University Hospital,

Tübingen, Germany



Finite elements for plates in bending

Degrees of freedom



4-node element
12×12 matrix

Displacements

Forces

3 DOF's per node: displacements w, rotations φ_x, φ_y | force F_z, moments M_x, M_y

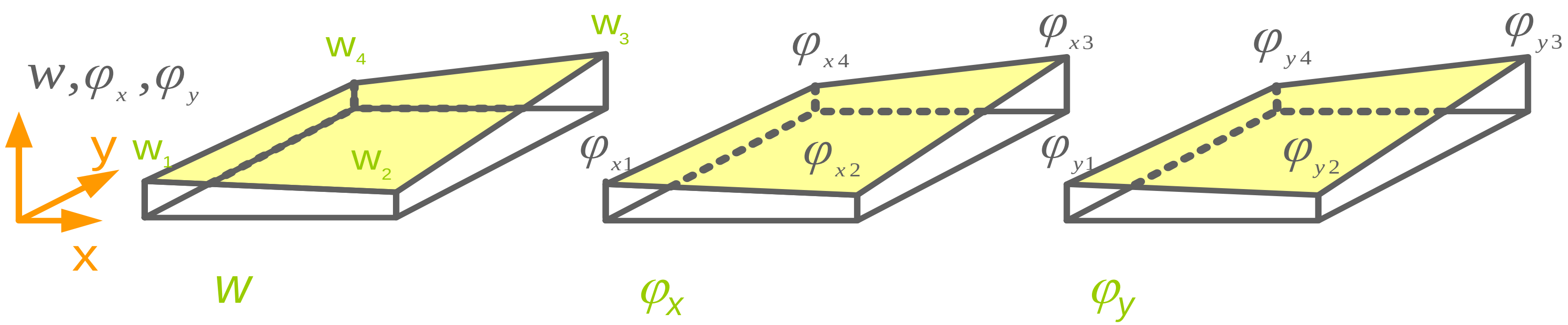
Element types

- Plate elements with shear deformations (Reissner-Mindlin plate theory)
- Plate elements without shear deformations (Kirchhoff plate theory)
- DKT/DKQ elements (diskrete Kirchhoff triangle/quadrilateral)
- Hybrid elements (shear rigid or shear flexible)
- Plate elements as „degenerated" solid elements

Finite elements for plates in bending

4-node plate element with shear deformations

Shape functions



Bilinear interpolation of the displacement w and the rotations φ_x and φ_y

Stiffness matrix

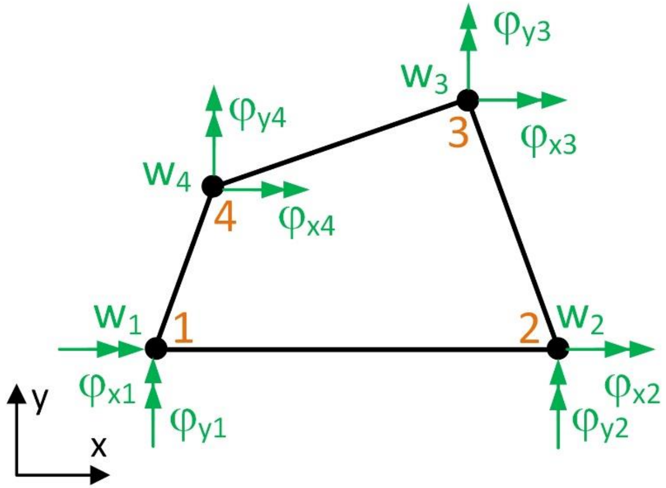
$$\underline{K}_e \cdot \underline{u}_e = \underline{F}_e \qquad \underline{K}_e = \int \underline{B}_b^T \cdot \underline{D}_b \cdot \underline{B}_b \, dx\, dy + \int \underline{B}_s^T \cdot \underline{D}_s \cdot \underline{B}_s \, dx\, dy$$

12 x 12 matrix — Bending — Shear

Finite elements for plates in bending

DKT-DKQ elements

- Quadrilateral shear rigid elements (Kirchhoff plate theory) based on displacement shape functions cannot be formulated due to mechanical and mathematical reasons.
- Formulation is based on shear flexible elements. But shear deformations at discrete points in the element are set to be zero as in the Kirchhoff plate theory. Hence DKT/DKQ elements are shear rigid elements.



DKT = Discrete Kirchhoff Triangle

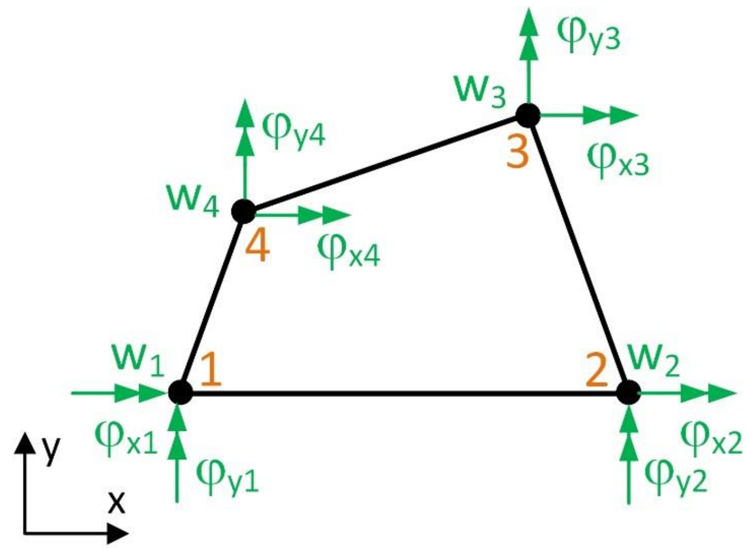
DKQ = Discrete Kirchhoff Quadrilateral

- efficient elements
- often implemented in FE programs

Finite elements for plates in bending

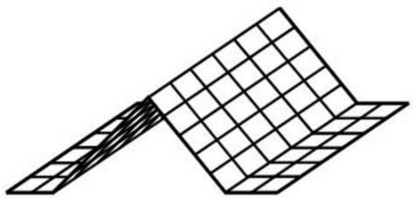
Hybrid elements

- Based on shape functions for internal forces inside the element and for displacments /rotations on the edges.
- Formulations for shear rigid and shear flexible plates.
- Different combinations of displacements and shape functions in the element area are possible.

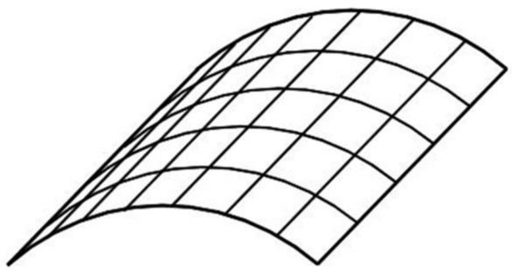


Finite elements for Shells

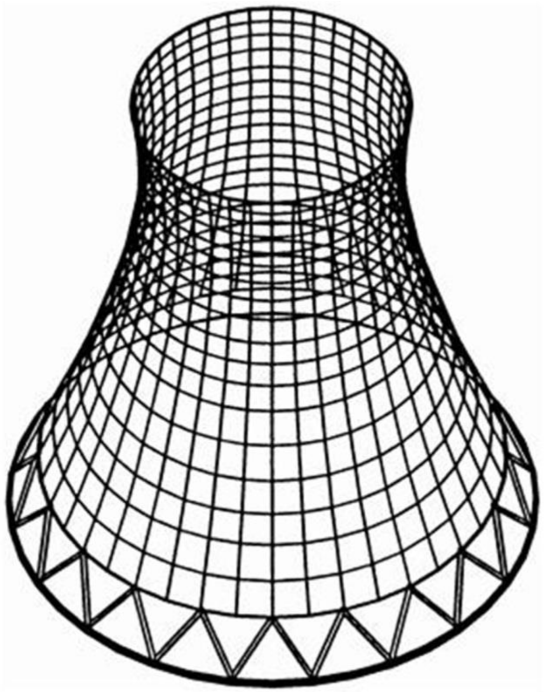
Plane shell elements



Folded plate



Simply curved shell



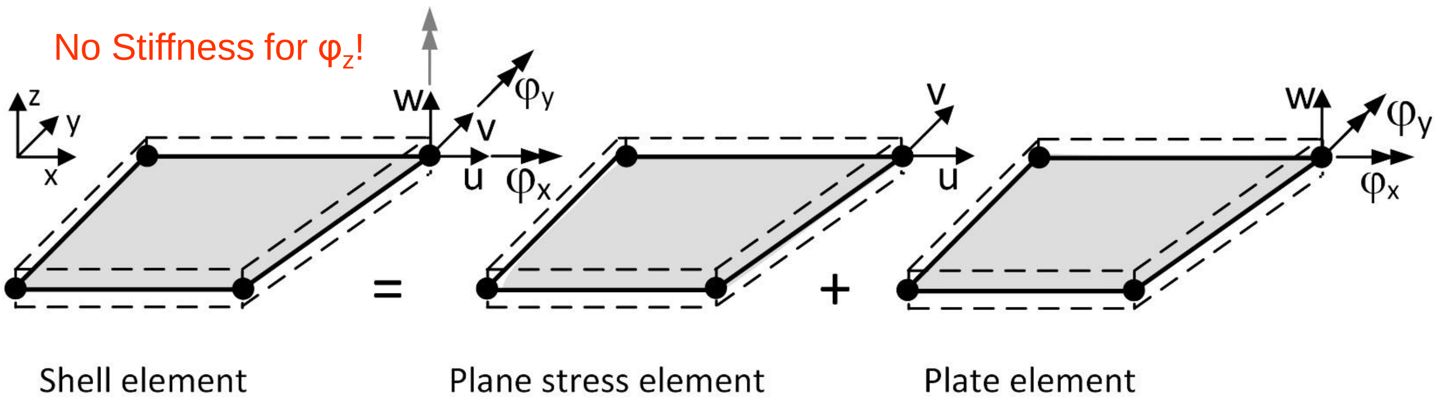
Double curved shell

Modeling with plane shell elements

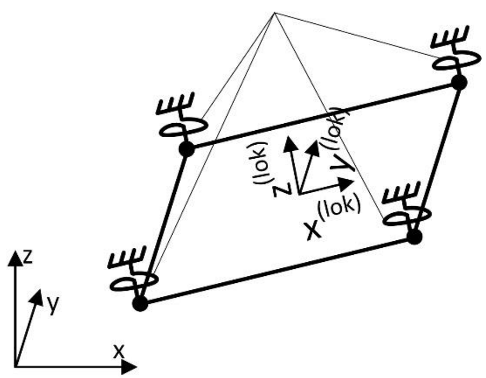
Finite elements for Shells

Plane shell elements

Composition of a shell element by a plane stress element and a plate element in bending



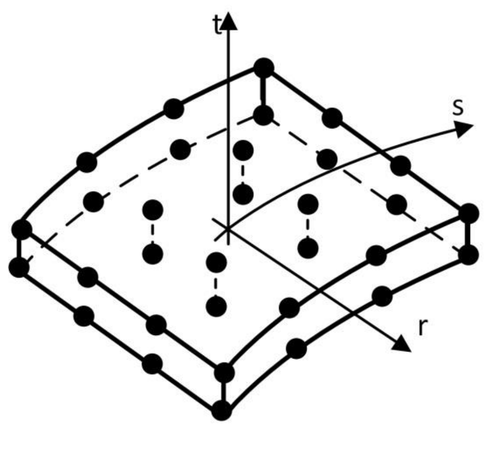
Rotational springs on an axis perpendicular to the element

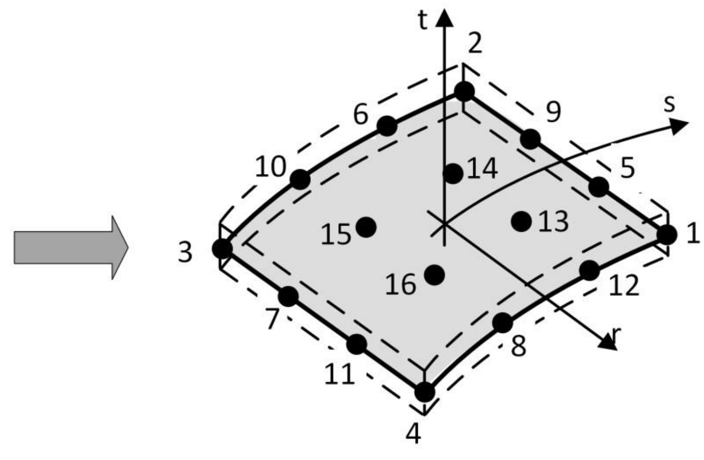


Artificial rotational springs are added to the stiffness matrix in order to avoid a singularity of the global stiffness matrix.
Spring constants are chosen to be very small (1/10000 of the smallest diagonal term of the stiffness matrix).

Finite elements for Shells

Curved shell elements :
Degenerated shell elements as modified volume elements





Conditions

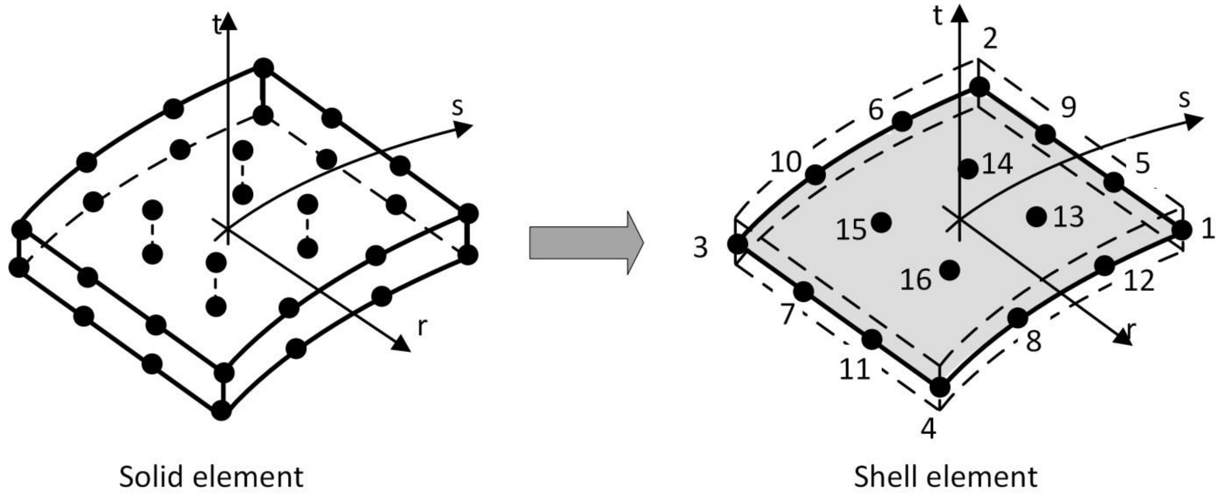
- Plate deformation: planes perpendicular to the plate remain straight and perpendicular to the middle plane (Bernoulli Hypothesis).
- The plate should not elongate normal to its plane.

New degrees of freedom

- The degrees of freedom of the nodal points can be expressed by the displacements and the rotations of the middle plane of the plate.

Finite elements for Shells

Curved shell elements :
Degenerated shell elements as modified volume elements

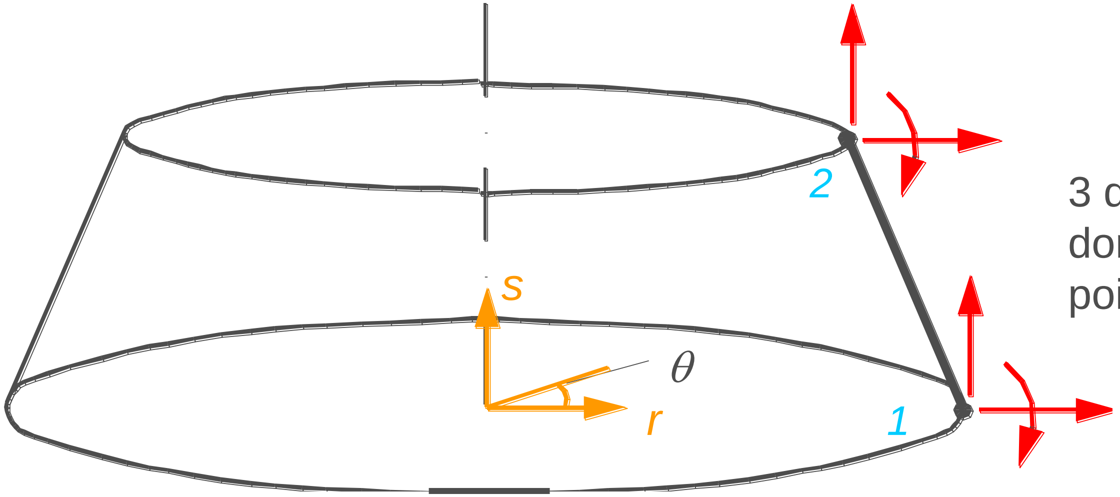


Properties

- These elements have the same properties as elements derived according to the theory of the shear flexible plate. This means that these shell elements must provide a strategy to deal with shear locking.
- With an isoparametric description of geometry these elements may also be curved.

Finite elements for Shells

Axisymmetric shell elements

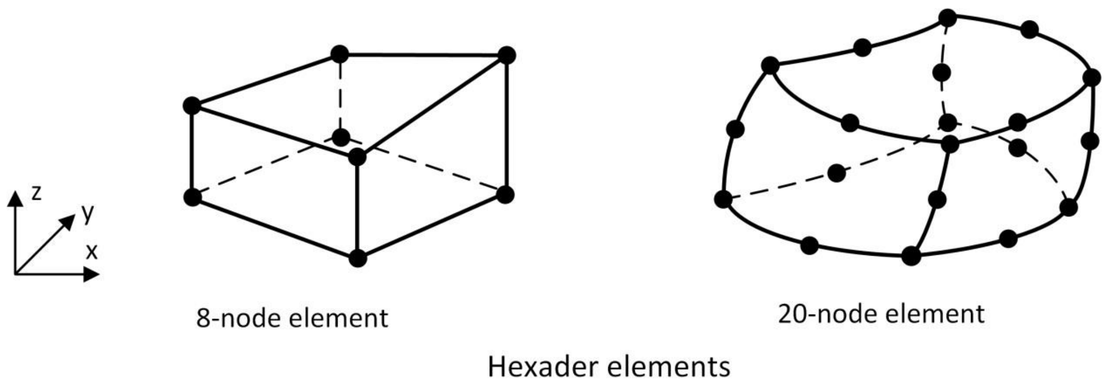


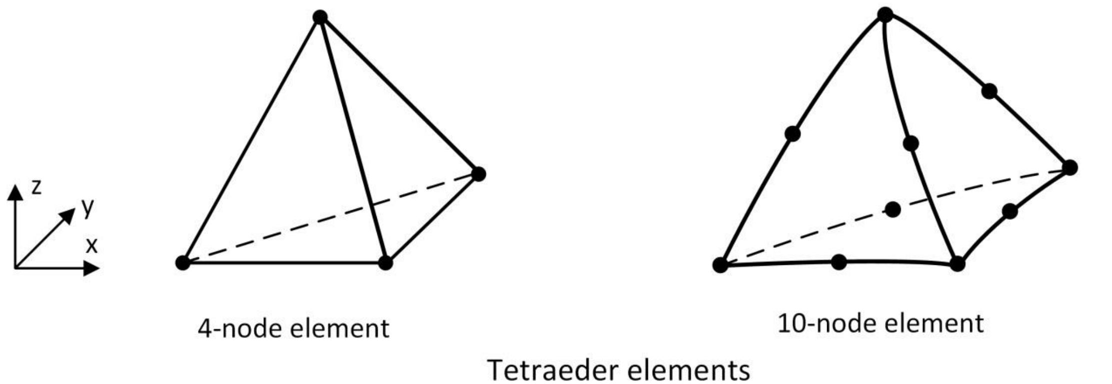
3 degrees of freedom at each nodal point

Axisymmetric shell element for axisymmetric loading

Finite elements for Solids

Threedimensional solid elements





Element types

Finite elements for Solids

Isoparametric threedimensional solid elements

Formulation of element types

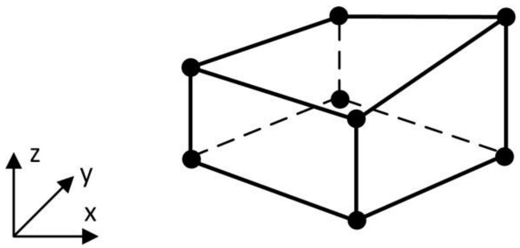
Shape (interpolation) functions for displacements $\underline{u} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$

Strains as derivitives of displacements

$$\underline{\varepsilon}^T = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \varepsilon_z & \gamma_{xy} & \gamma_{yz} & \gamma_{zx} \end{bmatrix} \qquad \underline{\varepsilon} = \underline{B} \cdot \underline{u}$$

Stresses

$$\underline{\sigma}^T = \begin{bmatrix} \sigma_x & \sigma_y & \sigma_z & \tau_{xy} & \tau_{yz} & \tau_{zx} \end{bmatrix}$$



3D-stress-strain relationship (Hook's law)

$$\underline{\sigma} = \underline{D} \cdot \underline{\varepsilon}$$

Stiffness matrix $\underline{K} = \int_V \underline{B}^T \cdot \underline{D} \cdot \underline{B} \cdot dV$

Finite elements for Solids

Isoparametric threedimensional solid elements

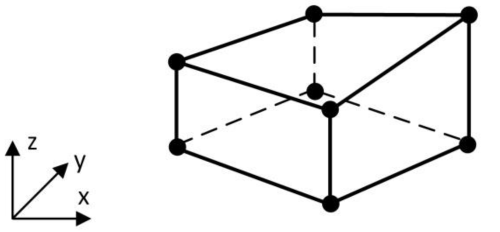
Properties

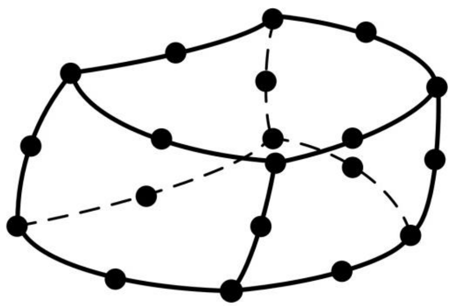
- Stiffness matrices of 3D solid elements are very large. For the 8-node cuboid a 24x24 matrix and for the 20-node element a 60x60 element stiffness matrix is obtained.

- The global system of equations may possess several hundred thousends of unknowns.

- Tetahedral elements as well as pyramid-, cone-, wedge- and prism-shaped volume elements with and without nodes on the element sides can also be formulated in this way as isoparametric elements.

- Three-dimensional solid elements with linear or bilinear shape functions can exhibit the same locking effects as two-dimensional membrane elements.

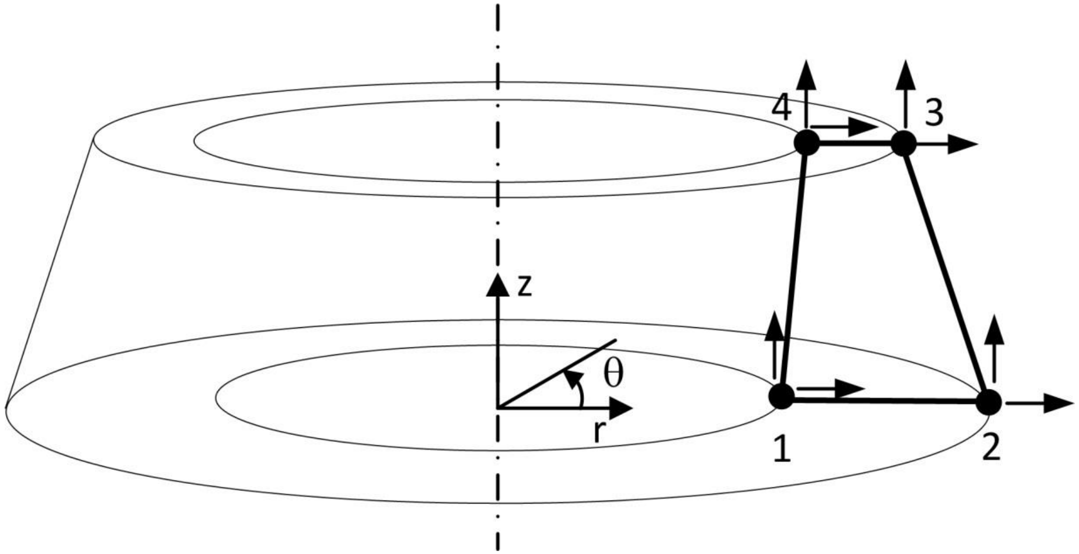




20-node element

Finite elements for Solids

Axisymmetric solid elements



2 degrees of freedom at each nodal point

Axisymmetric solid element for axisymmetric loading

- Allows to analyse axisymmetric threedimensional models as 2D models
- Stresses and displacements are axisymmetric

End

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